Design of geosynthetic-reinforced slopes in cohesive backfills

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ABSTRACT

Currently, geosynthetic reinforcements for slopes are calculated assuming the ground strength to be purely frictional, i.e. without any cohesion. However, accounting for the presence of even a modest amount of cohesion could allow using locally available cohesive soils as backfills to a greater extent and less overall reinforcement. But cohesive soils are subject to the formation of cracks that tend to reduce slope stability so their presence has to be accounted for in the design of the slope reinforcement. In the paper, limit analysis was employed to derive a semi-analytical method for uniform θ – θ slopes that provides the amount of reinforcement needed as a function of ground cohesion, tensile strength, angle of shearing resistance and of the slope inclination. Both climate induced cracks as well as cracks that form as part of the slope collapse mechanism are accounted for. Design charts providing the value of the required reinforcement strength and embedment length are plotted for both uniform and linearly increasing reinforcement distributions.

From the results, it emerges that accounting for the presence of cohesion allows significant savings on the reinforcement to be made, and that cracks are often significantly detrimental to slope stability so they cannot be overlooked in the design calculations.

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1. Introduction

Since the 1980s the use of geosynthetics with the aim of increasing the shear strength of cohesive soils has been investigated (Fourie and Fabian, 1987; Ingold, 1981; Ingold and Miller, 1983; Ling and Tatsuoka, 1994). In the 1990’s Zornberg and Mitchell in their review papers on cohesive backfills (Mitchell and Zornberg, 1995; Zornberg and Mitchell, 1994) state that the use of cohesive backfills has led to substantial savings in areas where granular materials are not locally available. More recently, substantial experimentation has been performed to investigate the behaviour of geotextile reinforced cohesive slopes (Hu et al., 2010; Noorazad and Mirmoradi, 2010; Wang et al., 2011). In particular non-woven geotextiles and geogrids have shown to be effective at increasing the strength of cohesive soils and improving drainage (e.g. Portelinha et al., 2013; Portelinha et al., 2014). However, in the methods currently available in the literature, reinforcements are still calculated assuming soils to be cohesionless (de Buhan et al., 1989; Jewell, 1991; Leshchinsky and Boedecker, 1989; Leshchinsky and Hanks, 1995; Michalowski, 1997). This conservative assumption is due to the fact that geosynthetics were initially conceived for cohesionless granular soils and that the first design guidelines published for geosynthetic reinforced earth structures disregard the beneficial effect of cohesion (e.g. Jewell, 1996). However, the recent edition of AASHTO LRFD bridge design specifications (AASHTO, 2012), allows for the inclusion of cohesion in the design of geo-reinforced slopes although unfortunately no formulae are provided for this purpose. The AASHTO revisit was prompted by the work of Anderson et al. (2008) which, for example, shows that an amount of cohesion as small as 10 kPa can reduce the thrust against an earth structure of up to 50–75% for typical design conditions. In light of these findings, Vahedifard et al. (2014) have investigated the beneficial effect of cohesion on geosynthetic reinforced earth structures based on limit equilibrium concluding that ‘the results clearly demonstrate the significant impact of cohesion on the Kae value’ (Kae being the design seismic active earth pressure coefficient). Unlike Vahedifard et al. (2014), this paper is concerned with the stability of geo-reinforced slopes in the absence of any retaining structure. One of the objectives of this

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In general, cohesive soils manifest limited, if not negligible, tensile strength so they are subject to the formation of cracks. The development of cracks in cohesive soils is due to weather action, e.g. cycles of wetting and drying (Take and Bolton, 2011). The determination of suitable values of cohesion and its degradation over time are discussed in detail in section 4.

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the influence of water pore pressures will be investigated.

2. Methodology

There are two main approaches to investigate the stability of geosynthetics-reinforced slopes, one where the local equations of equilibrium for an equivalent continuum formed by ground and reinforcement together are derived via homogenization techniques (e.g. de Buhan et al., 1989; Sawicki, 1983), called continuum approach by Michalowski and Zhao (1995), and another one, to be used here, where ground and geo-reinforcement are considered as two separate structural components, called structural approach (Michalowski and Zhao, 1995). Limit analysis (LA) can be used with both approaches. For sake of completeness, note that Sawicki and Lesniewska (1989, 1991) provide upper bounds on the required reinforcement for \( c - \phi \) slopes by employing the continuum approach together with the static (lower bound) method of LA. However, their solutions do not account for the presence of cracks which may significantly reduce slope stability as it will be shown in this paper, so their bounds cannot be relied upon to design the reinforcement. Instead Leshchinsky et al. (1986) used the structural approach together with variational limit equilibrium to determine the required reinforcement in \( c - \phi \) backfills. They claim the solutions found to be rigorous upper bounds in the sense of limit analysis, although this is not mathematically demonstrated in the formulation presented in their paper. Moreover, their solutions, provided for the case of a vertical slope only, suffer from neglecting the presence of cracks and overlooking some possible failure mechanisms.

In this paper the structural approach will be employed together with the kinematic (upper bound) method of LA to obtain lower bounds on the required level of reinforcement extending the LA formulation of Michalowski (1997) for purely frictional backfills to cohesive frictional (\( c - \phi \)) backfills. Note that LA assumes a simplified constitutive behaviour for both ground and reinforcement, i.e. rigid – perfectly plastic, and the validity of the normality rule, i.e. associated plastic flow, which at rigour does not hold true for most soils. A comprehensive treatment of limit analysis assumptions and limitations and their implications for slope stability can be found in (Chen, 1975).

3. Formulation of the problem

Geosynthetic reinforced slopes are subject to three main possible failure modes: reinforcement rupture, pull out failure, and direct sliding. In this paper, a rupture failure will be assumed in order to design the minimum amount of geo-reinforcement whereas a combined failure (rupture and pullout) will be assumed in order to calculate the required length of reinforcement.

Traction-free uniform \( c - \phi \) slopes with an inclination angle \( \beta \), ranging from 40° to 90° and reinforced with geosynthetic layers are here considered. A common choice for the distribution of reinforcement with depth is to employ reinforcement layers of equal strength laid at equal spacing or at a spacing decreasing linearly with depth. The former case gives rise to a uniform distribution (UD) of tensile strength over depth (see Fig. 1a) which can be expressed as:

\[
K_t = \frac{nT}{H}
\]

with \( K_t \) being the average strength of reinforcement in the slope, \( n \) the number of reinforcement layers, \( T \) the strength of a single layer at yielding point and \( H \) the slope height. Note that the influence of the overburden stress on the strength of the geosynthetics has been neglected for sake of simplicity (Michalowski, 1997). Instead, the second case gives rise to a linearly increasing distribution (LID) of strength over depth (see Fig. 1b):

\[
K = 2K_t \frac{(H - y)}{H}
\]

with \( K \) representing the local reinforcement strength in the slope, and \( y \) the vertical upward coordinate departing from the slope toe (see Fig. 2a). Note that there is plenty of evidence from field observations and experimental tests showing the load distribution in the reinforcement for slopes under working stress conditions is non linear (Allen and Bathurst, 2015; Yang et al., 2012; Viswanadham and Mahajan, 2007; Zornberg and Arriaga, 2003) so neither a UD nor a LID. However, the assumption of UD or LID is consistent with the LA assumption of the georeinforced slope being at impending failure and of rigid – perfectly plastic behaviour for the materials of the system (ground and reinforcement) which possess infinite ductility. These two assumptions imply that the distribution of forces in the reinforcement must coincide with the distribution of reinforcement strength (Michalowski, 1997).

Experimental tests in the centrifuge provide clear evidence that georeinforced slopes fail because of a rotational mechanism (Zornberg et al., 1998; Viswanadham and Mahajan, 2007; Yang et al., 2012) which is the mechanism here assumed: block E-B-C-D rotating around point P whose location is yet to be determined (see Fig. 2a). In this mechanism all deformations occur along the log-spiral D-C whose mathematical expression is:

\[
r = r_x \exp[\tan \phi (\theta - \chi)]
\]

where \( \theta \) and \( \chi \) are the angles made by \( r \) and \( r_x \) respectively with the horizontal axis, \( r \) is the distance between the spiral centre, point P, and a generic point on the log-spiral slip surface, and \( r_x \) is the length of the chord P-F. The deformations undergone by the reinforcement layers along the log-spiral slip surface and along crack B-C are illustrated in Fig. 2b and c respectively. The analysis here performed is a two dimensional analysis, i.e. plane strain conditions are assumed. Recently Zhang et al. (2014) and Gao et al. (2016) considered three dimensional failure mechanisms for reinforced slopes, the former employing limit equilibrium while the latter LA. Their analyses confirm that the most critical mechanisms are found for plane strain conditions.

Although from a physical viewpoint, the formation of cracks in cohesive slopes is due to the same mechanical cause, i.e. the presence of tensile stresses exceeding the ground tensile strength, here cracks will be grouped into two types according to the way they are dealt with by limit analysis, namely climate induced multiple cracks existing in the slope prior to the formation of any failure mechanism, here termed ‘pre-existing’ cracks, and cracks forming as part of a slope failure mechanism, here termed ‘formation’ cracks. A formation crack forms as part of a failure mechanism which is made of a log-spiral surface (D-C in Fig. 2a) where soil fails.

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purely in shear and of a crack (B-C in Fig. 2a) where soil fails in tension/shear.

Climate induced cracks need to be considered for reinforced slopes in regions subject to high annual temperature fluctuations, e.g., regions subject to a continental climate with rigid cold winters and arid summers as in central Asia and North America, whereas in regions with a temperate climate cracks are much less likely to occur. So in regions subject to high temperature fluctuations, the presence of weather induced cracks cannot be overlooked since these cracks can make the slope significantly less stable (Michalowski, 2013; Utili, 2013) while in regions with a temperate climate, the geo reinforced slope may be assumed to be intact. In both cases, the possibility of cracks forming as part of the failure mechanism will be accounted for.

An important simplifying assumption concerns the geometry of the cracks which are here assumed to be vertical for sake of simplicity. According to some laboratory experimental evidence cracks in cohesive slopes may exhibit non-planar (curved) shapes (Hu et al., 2010), however the assumption of a planar vertical shape allows considering kinematically admissible failure mechanisms made of a rigid rotation (Utili, 2013; Michalowski, 2013) and is convenient from a calculation point of view. Also this choice is consistent with previous literature on slope stability where cracks in cohesive slopes are assumed as vertical (e.g., Terzaghi, 1943; Spencer, 1968; Baker, 1981) and there is field evidence supporting this choice too (Dyer et al., 2009).

4. Derivation of the semi-analytical solution

According to the kinematic theorem of LA, the highest (best) lower bound to the required reinforcement can be derived from the following energy balance equation:

$$\dot{D} = \dot{W}$$

where $\dot{D}$ and $\dot{W}$ are the internal energy dissipation rate and the external work rate respectively. $\dot{D}$ is here calculated as follows:
\[ D = D_{(B-C)} + D_{(r-B-C)} + D_{(C-D)} + D_{(r-C-D)} \]  

(5)

with \( D_{(B-C)} \) and \( D_{(r-B-C)} \) being the energy rates dissipated along B-C by soil and reinforcement respectively, and \( D_{(C-D)} \) and \( D_{(r-C-D)} \) the energy rates dissipated along the log-spiral C-D (see Fig. 2a) by soil and reinforcement respectively.

With regard to \( D_{(B-C)} \), if the crack B-C is a pre-existing crack, no energy is dissipated by the soil since the crack is already formed hence \( D_{(B-C)} = 0 \); conversely if the crack B-C forms as part of the failure mechanism, energy is dissipated for the crack to form hence \( D_{(B-C)} \neq 0 \) with the value of \( D_{(B-C)} \) to be calculated as a function of the soil tensile strength (Michalowski, 2013). Usually when limit analysis is employed, the Mohr-Coulomb (M-C) function is adopted as failure criterion. But plenty of experimental evidence shows that the tensile strength associated with the M-C criterion is a significant over-estimation of the tensile strength, here called \( \sigma_t \), of most soils. To partially remedy this shortcoming but still use the simple linear M-C criterion, a tensile cut off is commonly adopted. Michalowski (2013) instead proposed to modify the M-C criterion by adopting a non-linear function in the stress range where cracks are expected to form (see Fig. 3). This non-linear function is made by a stress circle defined as being tangent to the M-C linear function \( \tau = c + \sigma \tan \phi \) and having the minor principal stress \( \sigma_3 = \sigma_t \) equal to the soil tensile strength, \( \sigma_3 = -\sigma_t \), with tensile stresses assumed negative according to the soil mechanics sign convention. The adopted failure criterion, indicated by the solid curve in Fig. 4, lends itself to simple LA calculations (see Michalowski, 2013) and on the other hand accounts for the non-linearity of soil shear strength in the stress range where cracks are expected to form. The energy expended for the formation of a crack turns out to be (Michalowski, 2013):

\[
D_{(B-C)} = \frac{\sigma_t^2}{1 - \sin \phi} \left( \frac{\mu}{\tan \mu} \right)^2 \int \frac{1 - \sin \theta}{\cos^2 \theta} \, d\theta
+ \frac{\sigma_t}{1 - \sin \phi} \int \frac{\sin \theta - \sin \phi}{\cos^3 \theta} \, d\theta
\]  

(6)

with \( \mu \) being the angle made by segment P-B with the horizontal (see Fig. 2a), \( \sigma_t^{M-C} \) being the uniaxial compressive strength consistent with the M-C criterion (see Fig. 3). The two surfaces of the formed crack B-C are considered no-tension non-cohesive perfectly smooth (no friction) surfaces, therefore the angle \( \eta \) between the velocity vector of the mass of soil sliding away and the crack surface is \( 0^\circ < \eta < 180^\circ \) (see B-C in Fig. 2a).

It is convenient to introduce a dimensionless coefficient, \( t \), defined as the ratio of the ground tensile strength, \( \sigma_t \), to be measured experimentally, over the maximum unconfined tensile strength consistent with the M-C criterion, \( \sigma_t^{M-C} \) (see Fig. 3):

\[ t = \frac{\sigma_t}{\sigma_t^{M-C}} \]  

(7)

It is straightforward to observe that \( 0 \leq t \leq 1 \). Both \( \sigma_t^{M-C} \) and \( \sigma_t^{M-C} \) are uniquely related to \( c \) and \( \phi \):

\[ \sigma_t^{M-C} = 2 c \left( \frac{\cos \phi}{1 - \sin \phi} \right) \]  

(8)

The amount of cohesion and tensile strength that can be relied upon in the design of backfills made of \( c-\phi \) soils depends on several

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factors that vary over time, to name a few: the ground moisture content; the level of the phreatic line in the slope and the presence of suction above it; the intended design lifetime for the reinforced slope since this has implications on the weather induced deterioration the soil strength is likely to experience over time. Several publications deal with the choice of the values for $c$ and $\phi$ for clay soils with the use of peak strength, residual strength, operational strength (Potts et al., 1997) and critical state strength advocated depending on the geotechnical problem tackled. The choice of the value for $c$, $\phi$ and $t$ is outside the scope of this paper. Take and Bolton (2011) provide a good coverage of the literature with regard to such a choice for clay slopes. Here it is enough to recall that the designer must be careful to design the reinforcement considering the worst case scenario in terms of hydraulic conditions that can occur over the entire lifetime of the slope and adopting a cautious approach. Also it is recommended that cohesion and tensile strength of the backfill are periodically monitored during the lifetime of the slope to measure their progressive deterioration over time due to environmental action, e.g. drying – wetting cycles and freezing – thawing cycles.

It is important to note that even in case of soils possessing no true cohesion, i.e. exhibiting zero shear strength at zero confinement, their shear strength can still be suitably described by the failure criterion here adopted with $t = 0$ and $c \neq 0$ (see Fig. 4). In this case $c$ is to be interpreted as an apparent cohesion with the strength envelope intercepting the $r$ axis at the origin. From a mathematical point of view the presence of this apparent cohesion means that the straight part of the failure criterion is above the $r = \tan \phi$ line and therefore reinforcement can be saved. The lack of true cohesion (and of any tensile strength) for these soils will be reflected in the solution (and in the results obtained) by the onset of deep cracks.

Now substituting equations (7)–(9) into Eq. (6), the following expression is obtained for the energy dissipated in the ground due to the formation of a crack:

$$D_{\text{h}(\text{B-C})} = c \bar{r}^2 \left( \frac{\sin \chi}{\tan \mu} \right)^2 \left[ \frac{\cos \phi}{1 - \sin \phi} \int_{\phi}^{\mu} \frac{1 - \sin \theta}{\cos^2 \theta} d\theta \right. $$

$$+ \left. \frac{2t \cos \phi}{1 - \sin \phi} \int_{\phi}^{\mu} \frac{\sin \theta - \sin \phi}{\cos^2 \theta} d\theta \right]$$

$$= c \bar{r}^2 g_3(\chi, \nu, \zeta, \psi, \phi, t)$$

(10)

The energy dissipated by the reinforcement along the crack is unaffected by the type of crack, ‘pre-existing’ or ‘formation’, and is given by (Michalowski and Zhao, 1995):}

$$D_{\text{r}(\text{B-C})} = \int_{\text{B-C}} K \bar{u}_c \sin \eta \, dh$$

(11)

where $\bar{u}_c$ represents the velocity vector along B-C (see Fig. 2a) and $dh$ an infinitesimal length of the crack. They can be expressed as:

$$\bar{u}_c = r_c \theta = \frac{r_c \cos \zeta \cos \theta}{\cos \theta} \hat{\theta}$$

(12)

$$dh = \frac{r_c \, d\theta}{\cos \theta}$$

(13)

with $r_c$ being the distance between point P and a generic point along the crack. Substituting equations (12) and (13) into Eq. (11) and after integration, the following expression is obtained:

$$D_{\text{r}(\text{B-C})} = \int_{\text{B-C}} \frac{1}{2} K \bar{r}^2 \left[ \exp\{2 \tan \phi (\zeta - \chi) \sin^2 \zeta - \sin^2 \chi \} \right] \frac{1}{2}$$

$$= \frac{1}{2} K \bar{r}^2 g_4(\chi, \nu, \zeta, \psi)$$

(14)

The expression for the energy dissipated in the soil along the log-spiral part of the failure mechanism (log-spiral C-D in Fig. 2a), $D_{\text{h}(\text{C-D})}$, is provided by Chen (1975):

$$D_{\text{h}(\text{C-D})} = c \bar{r}^2 \exp\{2 \tan \phi (\zeta - \chi) \} \frac{\exp\{2 \tan \phi (\nu - \chi)\}}{2 \tan \phi}$$

$$= \bar{r}^2 g_1(\chi, \nu, \zeta, \psi, \phi)$$

(15)

where $\theta$ is the angular velocity of the sliding wedge, $\nu$ and $\zeta$ are the angles made by $r_c$ and $r_c$ with the horizontal line respectively.

The energy dissipated by the reinforcement along the log-spiral part of the failure mechanism, $D_{\text{r}(\text{C-D})}$, is calculated by integrating $dD_{\text{r}(\text{C-D})}$ over C–D. $dD_{\text{r}(\text{C-D})}$ is given by (Michalowski and Zhao, 1995):

$$dD_{\text{r}(\text{C-D})} = \frac{w}{\sin \eta} \int_{0}^{\lambda \bar{r}} K \sin \bar{c} \bar{c} \, dx = K \sin \lambda \bar{u} \cos(\lambda - \phi)$$

(16)

with $w$ being the width of the discontinuity band (see Fig. 2b), $\lambda$ the angle made by the reinforcement layer with the discontinuity surface, $\bar{c}$ the strain rate of the reinforcement layer in the longitudinal direction, and $\bar{u}$ the velocity vector of the sliding ground. For sake of space, calculations are here reported only for the case of UD of reinforcement (i.e. $K = K_r$), while calculations for LID reinforcements are reported in Appendix A. The energy dissipated by the reinforcement over the log-spiral part (C-D) is (Michalowski and Zhao, 1995):

$$D_{\text{r}(\text{C-D})} = \int_{\text{C-D}} K \sin \lambda \bar{u} \cos(\lambda - \phi) \frac{r \, d\theta}{\cos \phi}$$

(17)

After integration, the following expression is obtained:
\[ D_{(C-D)} = \frac{1}{2} K_t \dot{\theta}_r^2 \exp[2\tan(\phi - \chi)]\sin^2 \nu - \exp[2\tan(\zeta - \chi)]\sin^2 \zeta \]
\[ = K_t \dot{\theta}_r^2 g_2(\chi, \nu, \zeta, \phi) \]

Note that the reinforcement layers lying above the centre of rotation \( P \) are subject to compressive stresses and therefore buckling, hence they are discarded in the calculation of \( D_t \) (Michalowski, 1997).

From Eq. (14) and Eq. (18) it emerges that the energy dissipated by the reinforcement along the spiral part \( F-C \) for the case of intact (un-fissured) slope is the same as the energy dissipated by the reinforcement along the crack (\( B-C \)), i.e. \( D_{(F-C)} = D_{(B-C)} \). This means that the energy dissipated by the reinforcement is not affected by the presence, or absence, of cracks.

External work \( (W_e) \) is done by the weight of the sliding wedge \( E-D-C-B \) \( (W_e) \) and any pore water pressure in the ground \( (W_w) \). The term \( W_e \) is here calculated as the work of block \( E-D-F \) minus the work of block \( B-C-F \) (Fig. 2a). The work of block \( E-D-F \) and of block \( B-C-F \) are calculated by the algebraic summation of the work of blocks \( P-D-F \), \( P-E-F \) and \( P-D-E \) (Chen, 1975) and of blocks \( P-C-F \), \( P-B-F \) and \( P-C-B \) (Utili, 2013; Utili and Nova, 2007) respectively. So

\[ W = W_1 - W_2 - W_3 - \left( W_4 - W_5 - W_6 \right) + W_w \]
\[ = \gamma \dot{\theta}_r^2 \left[ f_1 - f_2 - f_3 - f_4 + f_5 + f_6 + f_w \right] \]

The analytical expressions for \( f_1, f_2, f_3, f_4, f_5, f_6 \) and \( f_w \) are given in Appendix B. Note that here only static forces are considered for sake of simplicity. However, in case of seismic excitation, the formulation here presented can be straightforwardly extended to include seismic loads by adding the contribution of the seismic pseudo-static forces to the external work as shown in (Utili and Abd, 2016).

Substitution of the various energy rate contributions calculated into the energy balance equation (Eq. (4)), provides the objective function to be optimised to determine the required georeinforcement. Substituting Eq. (5) and Eq. (3) with their components into Eq. (4) and rearranging, \( K_t \) is determined as:

\[ K_t = \frac{(f_1 - f_2 - f_3 - f_4 + f_5 + f_6 + f_w)}{\gamma H} \left( \frac{g_1 + g_2}{g_3 + g_4} \right) \]

\[ \frac{c}{\gamma H} \left( \frac{g_1 + g_2}{g_3 + g_4} \right) \]

Eq. (20) provides an expression of general validity covering both types of cracks: pre-existing and formation cracks. In the following, first the case of georeinforced intact slope is treated followed by the case of slopes exhibiting cracks.

4.1. Intact slopes

Immediately after construction the reinforced slope can be thought of as intact. The unconstrained minimisation of \( f \) over the three geometrical variables \( \chi, \nu, \zeta \) provides the least (best) lower bound on the required level of reinforcement, \( K_t/H \). The failure mechanism is identified by the values of \( \chi, \nu, \zeta \) associated with the found lowest lower bound. Length and location of the crack which forms as part of the failure mechanism are found as a result of the minimisation. In Fig. 5, the level of reinforcement required is plotted for various slope features. The results are commented on in section 5.

4.2. Slopes manifesting (pre-existing) cracks

As observed earlier on, several cracks may develop over time in a georeinforced slope due to weather actions. Among these cracks the failure mechanism will always engage the one crack that has the most adverse effect on stability. There may also be the situation of the failure mechanism not engaging any existing crack. This can happen depending on the location and depth of the cracks. Utili (2013) analysing unreinforced slopes shows that only cracks in a (central) zone of the slope will be engaged by the slope failure

![Fig. 5. Required reinforcement versus soil cohesion for a slope with \( \phi = 20^\circ \) (a) uniform distribution of reinforcement, (b) linearly increasing distribution. Grey lines indicate the constraint of maximum crack depth is active, while black lines indicate the constraint is inactive. The mark + signals the boundary between the two.](image-url)
mechanism. The worst case scenario for the stability of the slope is found by setting $D_{i(B-C)} = 0$ in Eq. (20), to reflect the fact that no energy is dissipated by crack formation:

$$K_t = \frac{(f_1 - f_2 - f_3 - f_4 + f_5 + f_6 + f_w)}{\gamma H} = f_{\text{deep pre-existing}}(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H)$$

and minimising $f_{\text{deep pre-existing}}(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H)$ over the three angles $x,v,\zeta$. $f_{\text{deep pre-existing}}(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H)$ is a particular case of $f(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H,t)$ in Eq. (20), and is independent of the ground tensile strength. The values of $x,v,\zeta$ identify the geometry of the most adverse failure mechanism for the slope with the angle $\zeta$ identifying the crack most adverse to the stability of the slope. However, it is unlikely that such an adverse crack will ever be present, but instead the multiple cracks induced by weather action over time in the slope will be less critical. Assuming the existence of the most adverse crack implies that the very worst case scenario in terms of weather induced cracks is assumed which can be a desirable choice for a conservative design, so the minimisation of $f_{\text{deep pre-existing}}(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H)$ provides a conservative estimation of the required reinforcement for slopes subject to weather induced cracks. If the designer instead wishes to make a less conservative and more realistic assumption, an equality constraint prescribing either depth or location of the cracks or both can be added into the search of the least lower bound turned out to be a failure mechanism.

4.3. Maximum depth of cracks

The maximum depth for a crack which is part of a failure mechanism has to be limited due to the requirement that the new slope profile left after failure has occurred has to be stable (the new vertical slope on the right of B-C in Fig. 2). In theory cracks deeper than this maximum depth may form, but if they become part of a slope failure mechanism, the mechanism will engage them above their bottom tip so that the engaged crack depth will be less than or equal to the maximum depth. Lower and upper bounds obtained by LA to the maximum crack depth, $h_{\text{max}}$, were first proposed by (Terzaghi, 1943; Spencer, 1967) and (Spencer, 1968; Michalowski, 2013) respectively. Here to stay on the side of caution, an upper bound rather than a lower bound was prescribed. In this case of a dry crack takes the following expression (Michalowski, 2013):

$$h_{\text{max}} = \frac{3.83 c \tan \left(\frac{\pi}{4} + \frac{\phi}{2}\right)}{\gamma}$$

4.4. Mechanisms passing above the slope toe

Failure mechanisms may in principle daylight on the slope face above the slope toe (Utili, 2013). So mechanisms passing above the toe were considered in our analysis for both types of reinforcement distribution by discretising the slope face in several points and calculating the stability factor associated to each mechanism. In all the cases considered no mechanism passing above the slope toe turned out to be a failure mechanism.

5. The minimum required reinforcement

The lower bounds on the required reinforcement expressed in dimensionless form, $K_t/\gamma H$, obtained by the minimisation of $f(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H)$ and of $f_{\text{deep pre-existing}}(x,v,\zeta,\beta,\gamma w/\gamma,\phi,c/\gamma H,t)$ plotted in Fig. 5 against an assigned level of soil cohesion for the case of intact slopes and of slopes manifesting pre-existing cracks respectively. The charts obtained for $\phi = 20^\circ$ cover the whole spectrum of cohesive geomaterials ranging from $c = 0$, for cohesionless materials (e.g. a granular fill), to values of cohesion so high that no reinforcement is needed (where the lines intersect the horizontal axis). Note that at $c = 0$ all the three lines depart from the same point since in case of zero cohesion, no cracks can form and the obtained $K_t/\gamma H$ values coincide with the values already published in the literature for purely frictional fills as it can be expected (e.g. Michalowski, 1997). Grey coloured lines indicate that the constraint on the maximum crack depth was active, whereas black lines indicate that the constraint was inactive.

From the charts emerges that the three lines tend to diverge for increasing cohesion. This trend can be explained by considering the term for the energy dissipated by the ground along the crack, $D_{i(B-C)}$ the higher the value of cohesion, the higher is the influence of $D_{i(B-C)}$ in the energy balance equation (see Eq. (20)) so the larger is the difference between the case of slopes subject to the most adverse pre-existing crack ($D_{i(B-C)} = 0$) and of intact slopes subject to crack formation ($D_{i(B-C)} \neq 0$). In the latter case, higher values of
cohesion also imply a larger influence of the ground tensile strength on slope stability (see the lines for \( t = 1 \) and \( t = 0 \) in Fig. 5) due to the term \( \frac{1}{1 - \sin \frac{\theta}{2}} \left( \frac{\sin \frac{\theta}{2}}{\sin \frac{\phi}{2}} \right)^{\frac{2}{1 - \sin \frac{\phi}{2}}} \tan \theta \frac{\sin \theta - \sin \phi}{\cos^2 \theta} d\theta \) in the analytical expression of \( D_{s,B,C} \) (see Eq. (10)). Slopes subject to the most adverse pre-existing crack require significant more reinforcement (because they are less stable) than intact slopes especially in case of steep slopes with a UD of reinforcement and low \( \phi \). Also note that whatever the crack scenario is, LID reinforcements are more effective (i.e. lower required overall reinforcement) than UDIs of reinforcement because more reinforcement layers are concentrated in the lower part of the slope.

5.1. Numerical validation

The validation exercise undertaken entailed both finite element displacement-based analyses with strength reduction technique (FESR), where the validity of the normality rule \( (\phi = \psi) \) consistently with the theory of limit analysis was assumed and Finite Element Limit Analyses (FELA). Slopes of various inclinations reinforced with a UD of reinforcement were considered. The presence of the most adverse crack to slope stability was assumed. All the simulations were performed using the software package Opt+umCE (OptumCE, 2014). The location and depth of the most adverse pre-existing cracks found by minimisation of Eq. (20) for various values of \( c/\gamma H \) were an input data to both the FESR and FELA simulations. The pre-existing cracks were implemented as joints of negligible friction but no tensile strength.

The obtained values of \( K_t/\gamma H \) are plotted in Fig. 6b against \( c/\gamma H \). It can be noted that the analytical LA upper bounds found by minimisation of Eq. (20) are slightly better than the FELA upper bounds. This finding is consistent with previous literature (Loukidis et al., 2003; Utili and Abd, 2016) showing that the analytical upper bounds found assuming a rigid rotational mechanism are lower (better) than the FELA upper bounds. Also the difference between the analytical upper bounds and the FELA lower bounds is lower than 14% for any value of cohesion considered. Therefore, true collapse values can be determined with an accuracy of \( \pm 7\% \) by taking the average of the two bounds. Moreover, the values of \( K_t/\gamma H \) determined by FESR simulations are found to be very close to the analytical upper bounds. Therefore, it can be concluded that the validation exercise here presented supports the adoption of the analytical upper bounds for design purposes.

5.2. Charts for dry slopes

In Fig. 7 four design charts have been produced where \( K_t/\gamma H \) is plotted against slope inclinations ranging from 40° to 90° for various combinations of values of shearing resistance angle, cohesion and tensile strength of engineering interest as well as the case of the most adverse pre-existing crack being present.

Considering the case of intact slopes, it can be observed that for relatively low values of cohesion, \( c/\gamma H = 0.05 \), the tensile strength, \( t \), possesses a negligible effect on the required reinforcement level. But for higher levels of cohesion \( (c/\gamma H = 0.1) \), it becomes important: for instance for \( t = 0.2, 0.2 \) and 0.5 extra reinforcement amounts of 32%, 15% and 5% respectively are required over what needed in case of \( t = 1 \). Since it is difficult to reliably determine the in-situ soil tensile strength, the charts in Fig. 7 can be used to decide whether the investment is worthwhile - for instance, if a soil exhibits low cohesion, spending money to determine \( \sigma_t \) is not worthwhile since \( \sigma_t \) would make very little difference to the required reinforcement; vice-versa for soils exhibiting high cohesion, proving the existence of some tensile strength would allow making important savings on the reinforcement. Finally, it is observed that the beneficial influence of some tensile strength is larger in slopes of high \( \phi \) and reinforced with a LID of reinforcement.

Considering now the case of the most adverse pre-existing crack being present in the slope, from Fig. 7 it can be noted that \( K_t/\gamma H \) becomes significantly larger for soils manifesting high values of cohesion and \( \phi \). To put this result in context, let us recall that this is a worst case scenario to be assumed when no other information about weather induced cracks is available and a conservative design is desired. If depth or location of the cracks can be ascertainment, a less conservative estimate of the required reinforcement can be obtained by imposing a crack depth or location as an equality constraint to be added into the search for the minimum of \( J_{\text{deep pref pre-existing}}(x, \psi, \phi, \beta, c/\gamma H, \gamma W/\gamma') \).

5.3. Illustrative examples

To showcase quantitatively the beneficial effect that accounting for cohesion may have in the design of geo-reinforcements, two design examples are considered.

- Example (1) - design the reinforcement to stabilise a steep clay slope 8 m high with 75° inclination on the horizontal with the clay exhibiting \( \phi = 20\% \), a modest cohesion of 7.5 kPa and a unit weight of 18.5 kN/m³.
- Example (2) - design a 5 m high embankment 45° inclined to be built in a continental climate region (presence of pre-existing cracks) utilising a cohesive backfill weighing 20 kN/m² and with a shearing resistance of \( \phi = 20\% \), 5 kPa of apparent cohesion but no tensile strength.

The values of \( K_t/\gamma H \) for the two examples are reported in Table 1. From the table emerges that accounting for the beneficial effect of cohesion allows saving of up to 82% in example 1 and up to 40.8% in example 2.

5.4. Influence of pore water pressure

The effect of various hydraulic conditions on the required level of reinforcement is here analysed by employing the so-called \( r_u \) method (Bishop and Morgenstern, 1960). This is an approximate method to account for the presence of pore water pressure in partially saturated slopes. Here, a uniform value of \( r_u \) is assumed throughout the entire slope and an effective stress analysis is carried out. The depth of water within the crack was calculated to be consistent with the assumed value of \( r_u \) and the maximum depth of crack was chosen consistent with the seepage scenario examined according to table 2 of Michalowski (2013). In Fig. 8 values of \( K_t/\gamma H \) are plotted against slope inclinations ranging from 40° to 90° for \( r_u = 0, 0.25 \) and 0.5. Looking at the charts two important observations can be made: the effect of the presence of cracks is higher in UDIs of reinforcements especially for steep slopes and the destabilising influence of pore water pressure is significantly higher in UDIs of reinforcement than in LIDs. The reason for this is that in case of a LID, more reinforcement layers are laid in the lower part of the slope.
5.5. Shallow (pre-existing) cracks deepened by the failure process

In this section we consider the possibility of a failure mechanism entailing the extension of a pre-existing crack underneath its bottom tip (point I in Fig. 9a) as part of the failure process. This implies that energy is dissipated underneath the crack tip, i.e. $D_s(I/C_0) > 0$. The minimisation of $f(c, y, z, f, b, c/g_H, t, g_w)$ (see Eq. (20)) over $c, y, z$ constrained by the following additional equation prescribing the pre-existing crack depth, $h_{pre-existing}$:

$$
\exp(\tan \phi \xi) \sin \xi = \exp(\tan \phi \chi) \sin \chi \left(1 - \frac{h_{pre-existing}}{H}\right) + \frac{h_{pre-existing}}{H} \exp(\tan \phi \nu) \sin \nu
$$

(23)

specifies the amount of reinforcement needed having assumed the presence in the slope of the most adverse crack among those of depth $h_{pre-existing}$. The horizontal position of the crack engaged by the failure mechanism is provided as a result of the optimization process.

In Fig. 9b, values of $K_r/g_H$ are plotted against prescribed pre-existing crack depths for different values of $c/g_H$. The grey lines refer to failure mechanisms involving further crack formation ($D_s(I/C_0) > 0$), whereas the black lines refer to failure mechanisms not involving further crack formation ($D_s(I/C_0) = 0$ and $I = C$). For shallow pre-existing crack depths (small values of $h_{pre-existing}/H$) the grey lines are distinct from the black lines lying above them. This means that if crack formation due to the exceedance of the tensile strength is accounted for in the calculations, the failure mechanism is more critical than the failure mechanism found by

Fig. 7. Required reinforcement for intact slopes not subject to crack formation ($t = 1$), intact slopes subject to crack formation (limited tensile strength of $t = 0.5, t = 0.2$ and $t = 0$) and cracked slopes. (a) & (b) are for $c/g_H = 0.05$ while (c) & (d) are for $c/g_H = 0.1$. Grey lines indicate the constraint of maximum crack depth is active, while black lines indicate the constraint is inactive. The mark + signals the boundary between the two.
disregarding it. So the possibility of further crack formation cannot be overlooked and the reinforcement design should be based on the grey lines. Instead, at high values of $h_{pre}/C_0 = H$, grey lines and black lines coincide, so the critical failure mechanism does not entail the deepening of pre-existing cracks which are therefore called deep cracks to indicate that no deepening occurs as result of the slope failure mechanism taking place. The boundary between shallow and deep pre-existing cracks can now be unambiguously identified as the point where the grey and black lines no longer coincide (see the square symbols in Fig. 9b).

Another important observation is about the fact that the required reinforcement increases with the depth of the pre-existing cracks, but only until a certain threshold value beyond which it remains constant (see the horizontal parts of the lines in Fig. 9b). For values of $h_{pre}/H$ smaller than the threshold, the log-spiral part of the failure mechanism (D-C) joins the pre-existing crack at its bottom tip whereas for values larger than the threshold, the log-spiral part of the failure mechanism joins the pre-existing crack above its bottom tip. Importantly observing Fig. 9b, we can conclude that the most adverse situation for the stability of slopes subject to weather induced cracks occurs for the failure mechanism found by the minimisation of the function $\text{deep pre-existing}(c, \psi, \phi, \beta, h/C, \gamma)$ in Eq. (21) which also provides the most adverse crack for the slope as a result of the minimisation. This failure mechanism does not entail any further crack formation.

### 6. Length of reinforcement

To calculate the minimum length of the reinforcement layers, a combined failure mode consisting of pull-out in some layers and rupture (tensile failure) in others, needs to be considered. The normalised length of reinforcement, $L_r/H$, is calculated following the procedure set by (Michalowski, 1997) extended to the case of $c/\phi$ soil slopes and accounting for the presence of cracks. Assuming all layers are of the same length, it turns out to be:

$$L_r/H = \frac{1}{K_t}$$

<table>
<thead>
<tr>
<th>Normalised cohesion</th>
<th>Uniform distribution</th>
<th>Linearly increasing distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (1) $c/H = 0$</td>
<td>$t = 1$</td>
<td>$t = 1$</td>
</tr>
<tr>
<td>Case (2) $c/H = 0.05$</td>
<td>$t = 0.1307$</td>
<td>$t = 0.1460$</td>
</tr>
</tbody>
</table>

**Example 1, Required reinforcement $K_r/H$**

| Case (1) $c/H = 0$ | $0.2211$ |
| Case (2) $c/H = 0.05$ | $0.1307$ |

**Savings for Example 1**

- $t = 0.408%$ for case (2)
- $t = 33.9%$ for case (1)

**Example 2, Required reinforcement $K_r/H$**

| Case (1) $c/H = 0$ | $0.1288$ |
| Case (2) $c/H = 0.05$ | $0.0231$ |

**Savings for Example 2**

- $t = 82%$ for case (2)
- $t = 78.8%$ for case (1)

*The most unfavourable crack scenario is assumed.*

---

**Table 1**

| Examples of savings on the reinforcement that can be achieved by accounting for the presence of cohesion and tensile strength. |
|---------------------|----------------------|-------------------------------|
| Normalised cohesion | Uniform distribution | Linearly increasing distribution |
| Case (1) $c/H = 0$  | $t = 1$              | $t = 1$                       |
| Case (2) $c/H = 0.05$ | $t = 0.1307$         | $t = 0.1460$                  |
| Case (1) $c/H = 0$  | $0.2211$ |
| Case (2) $c/H = 0.05$ | $0.1307$ |

**Fig. 8.** Comparison of the required reinforcement between intact and cracked slopes (with $\phi = 20^\circ$ and $c/\gamma H = 0.1$): (a) UD of reinforcement; and (b) LID. Grey lines indicate the constraint of maximum crack depth is active, while black lines indicate the constraint is inactive. The mark $+$ signals the boundary between the two.
with $K_t/\gamma H$ determined from the semi-analytical method expounded in the previous sections; $j$ being the number of layers pulled out; $z_i$ being the overburden depth of reinforcement layer $i$ which for gentle slopes it can be less that the depth $z_1$ of the reinforcement layer below the slope crest, $f_b$ the bond coefficient between soil and reinforcement and $n$, the number of reinforcement layers.

An optimization procedure was carried out to find the maximum value of $L_r$ over the variables $(\chi, \psi, \zeta)$ for an example slope with $n = 6$, $f_b$ was taken as 0.6 according to the latest report from the U.S. Federal Highway Administration (Berg et al., 2009). The results, presented in Fig. 10, show that in case of the most
adverse pre-existing crack being present the longest embedment length is required and the higher the soil tensile strength the shorter the required reinforcement length. This finding is consistent with the results reported in section 4.

7. Conclusions

A new semi-analytical method based on limit analysis for the design of geo-reinforcement in cohesive backfills was presented. The method provides lower bounds on the amount of required reinforcement which depend on three soil strength parameters: \( \phi \), angle of shearing resistance, \( c \), cohesion, and \( \sigma_t \), tensile strength. Several design charts were provided for both uniform and linearly increasing reinforcement distributions.

In the paper it is shown that 1) accounting for the presence of cohesion allows achieving a less conservative design so that significant savings on the overall level of reinforcement can be made; 2) there are several situations where the presence of cracks reduces significantly the stability of the reinforced slopes so that in general they cannot be neglected in the stability analysis performed to design the amount of reinforcement required; 3) the soil tensile strength has a significant influence on slope stability, in case of high levels of cohesion and angle of shearing resistance, but little influence otherwise.

A validation exercise was undertaken by means of both finite element lower bound analyses and finite element with strength terms of hydraulic conditions over the entire slope lifetime is considered by the designer and that cohesion and tensile strength of the backfill are periodically monitored during the slope lifetime to measure their progressive deterioration over time due to environmental action.

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Appendix (A)

For linearly increasing distribution (LID), the expression for the energy dissipated by the geosynthetics along the log-spiral part C-D can be expressed as follow:

\[
D_{l(C-D)} = 2K_t \hat{\theta} r^2_g (\sin 2\theta - \sin \theta) \left[ \frac{(1/3) \exp[3 \tan \phi (v - \chi)] \sin^3 v}{-\exp[3 \tan \phi (\zeta - \chi)] \sin^3 \zeta} + \frac{\sin \chi}{2} \left( \exp[2 \tan \phi (v - \chi)] \sin^2 v - \exp[2 \tan \phi (\zeta - \chi)] \sin^2 \zeta \right) \right]
\]

\[
D_{l(C-D)} = K_t \hat{\theta} r^2_g \left[ \sin \frac{\chi}{\cos^2 \theta \sec^2 \theta d\theta} \right]
\]

\[
D_{l(B-C)} = 2K_t \hat{\theta} r^2_g (\sin \chi; \phi)
\]

Appendix (B)

The final expressions of the components of the external work rate can be listed as follow:

\[
W_1 = \gamma \hat{\theta} r^2 f_1 (\chi; v, \phi)
\]

\[
W_2 = \gamma \hat{\theta} r^2 f_2 (\chi; v, \phi, \beta)
\]
The angle $\theta_1$ can be found from this equation:

\[ \exp(\tan(\phi_1 - \chi)) \cos \theta_1 - \cos \chi + l_1 r_x = 0 \]


