1. Introduction

Power system stability becomes very important with the interconnection among large scale power grids. Many power systems face the problem of small frequency oscillation in 0.2 – 1.5 Hz range due to system interconnection or heavy load condition. Several studies reveal that the poorly damped small frequency oscillations depend on power system stability which participates in the stability analysis of small signals. If the oscillations are not damped sufficiently, instability may happen and sometimes may result in system collapse. It is therefore necessary to inspect the low frequency modes and the damping coefficients of the raw signal for dynamic system security. For this purpose, several signal processing techniques were used in the past by various researchers in order to detect the low frequency modes. By using dynamic assessment, following supports are provided to the power system:

- Timely recognition of system problems, enabling cost minimization through maintenance based on performance and providing a security test on network loading.
- Improvement of scheduling, function and control schemes to allow excellent application of transmission capability.
- Real time determination of transmission limits.

In modern power system, the reliability and functioning of the set-up is highly affected by uneven inter-area oscillations. They provide hinderance for enhancement in power transfer over long transmissions distances. Further, due to the deregulation of the power market, the patterns of the power transfer has become more random and erratic. As a result of which threat of triggering...
inter-area oscillations has increased. Just like local oscillations, the estimation of small frequency inter-area oscillation is a tough task for the usual SCADA arrangement. So, wide area measurement systems (WAMS) provides on-line estimation of the network by Phasor Measurement Units (PMU) installation. The dynamic time-stamped measurements of currents, voltages and angle differences across the transmission line are provided by PMU. Further, these collected PMU data are given to the control office via adequate communication channel and the system dynamic behaviors can be determined. Thus, parameters such as frequency and damping can be estimated which characterizes the electromechanical modes of the system and given to the system operator in real-time and due to which unstable oscillations, timely warnings can be given and sufficient emergency protective measures can be taken. Nowadays Japan, Australia, China, USA and many European countries have implemented WAMS into their grid in conjunction with SCADA system. A superior technology infrastructure is thus developed by WAMS which combines measurement based information into the grid management system. The total scheme thus provides operational support, measurement facilities and data utilization. Communication of sampled data at the rate 30 times per second or more can be provided by WAMS which augments conventional supervisory control and data acquisition (SCADA) in which measurements are “refreshed” once every 4 s which is very slow rate. Presently WAMS is applied as a complementary system to improve the operator’s real-time “situational awareness” which is required for protected and reliable operation of the network. Also in WAMS, functions of metering devices (i.e. new and traditional) with the abilities of communication systems is provided for proper monitoring, operation and control of power system in large area. The overall ability of this particular combination is that entire system data can be acquired at the same time and place i.e. the control center. Further the collected data can be utilized by the WAMS functions properly. Synchronous measurement of power system dynamics and online assessment of systems dynamic characteristics by quick centralization is nowadays possible to get by the development of WAMS technology. Due to excellent synchronization of WAMS time, smart applications to power system monitoring is obtained.

In the mid-1980s, the introduction of Synchronized Phasor Measurement System (SPMS) into the power system has made possible the measurement of voltages, currents and calculation of the angle between them due to the accessibility of Global Positioning System (GPS) and sampled data processing techniques. The sampling clock of SPMS uses GPS time to combine measured voltage phasors, currents phasors and time stamped phasors or MPUs. Normally PMUs are placed at isolated areas which compute voltage phasors, currents phasors and time stamped phasors received from GPS. A PDC collects information from the PMUs, discards redundant information, and aligns the time stamps. Communication system of SPMS is liable for data release between PMUs and PDCs. The PMU is a mechanism based on microprocessor which applies the skill of digital signal processors for measurement of 50/60 Hz AC voltage and current waveforms at a usual rate of 48 samples per cycle (2400/2880 samples per second). For this, the analog waveform for each phase are synchronized by Analog to Digital converter. For providing synchronous clock for the whole network, the time from GPS satellites are taken as input for a phase-lock oscillator and thereby waveforms of the total system are sampled with a precision of 1 microsecond. Thereafter calculation of current and voltage phasors are done by PMU which uses digital signal processing techniques. The measured phasors are collected by GPS time stamps and are transmitted to a PDC at the rates 30–60 samples per second [1].

A number of research activities in the countries like USA, Australia, China, Japan and many other European countries have been carried out by the researchers on the issue of small signal stability and wide area measurement systems (WAMS) [2–4]. In recent years these countries have integrated WAMS into their grid next to the SCADA system [5–10]. In order to precisely investigate inter-area oscillations, quite a few linear analysis techniques have been reported. Among them, Kalman filter and Prony analysis have been assigned and formerly operating in almost all national grids [9–11]. Real incident of synchronized multiple phasor measurements in Northern Mexico are taken to inspect the advantages of nonlinear time series schemes to illustrate the temporal qualities of complex nonlinear, nonstationary oscillations [12]. Prony analysis is a upcoming technique that is a modification of Fourier analysis by directly inspecting the low frequency present in a given signal [12]. In [12] Prony analysis is applied to take out the frequency information from large-scale system tests or disturbances. To detect poorly damped oscillations, WAMS in combination with Kalman filter is proposed in [13,14]. The method in [13,14] gives good results for detection, however more rigorous tasks on real field information are required to further know the numerical properties behavior of the method. A system with online global monitor of the western Japan 60 Hz power system [15] is being developed by researchers known as campus wide area measurement system (campus WAMS). Campus WAMS consisted of data servers and commercialized PMUs [16] which constantly rectify the practical information by a campus information network.

A scheme is proposed in [17] for the inspection and detection of small frequency oscillations. In this scheme [17], an improved Prony analysis is presented which inspects the concerned oscillatory modes quickly and correctly. A Discrete Fourier Transform (DFT) based technique for the parameter detection of small frequency oscillating signal is reported in [18]. Estimation of parameters like frequency, amplitude, damping factor and phase by fitting a damped cosine function with a discrete Fourier spectrum is reported in [18] where the estimation error is minimized by detecting low frequency from the peak amplitude and damping factor. Fast computation speed is reported in [18]. Analysis of wide area phasor measurement unit (PMU) using Hilbert analysis is reported in [19] where the characterization of the non-linear oscillations from co-ordinated wide-area measurements are being done by Hilbert method which supply a number of insights into the nature of temporal spectral variations. However due to the effect of numerical errors, further extension and refinement in the algorithm of [19] is needed. Identification and monitoring of small frequency oscillations on the PMU’s and wavelet technique is discussed in [20]. The scheme in [20] gives better insight for low frequency estimation from PMU data, however from the contour plot the instantaneous frequencies cannot be detected.

This paper presents a practical case study in which investigations were carried out with real time wide area measurement systems data collected from four different Phasor Measurement Unit (PMU) i.e. Dadri, Vindyachal, Kanpur and Moga which were located near the recent disturbance event at the Northern Grid. Low frequency modes were estimated from the ringdown portion of the collected signal. The main objective of the paper is to detect correctly the low frequency oscillatory modes using wide area measurement signal by implementing methods based on signal processing techniques. Signal processing methods like Fourier Transform (FFT), Prony analysis (PA), S-transform (ST), Wigner Ville distribution (WVWD). Estimation of Signal Parameters by Rotational Invariance Technique, Hilbert-Huang Transform and Matrix Pencil Method were used to detect the low frequency modes. Simulation results from the analysis show that the seven
signal processing based techniques (PA, FFT, ST, WD) discussed in this paper provides satisfactory performance in detecting the low frequency modes.

The remaining sections of the paper are placed as follows. Description of real time wide area measurement systems (WAMS) data is discussed in Section 2. Section 3 provides the description of the seven signal processing based technique to detect the low frequency modes. Section 4 shows the simulation output and another case study of estimating low frequency oscillatory modes of Eastern Interconnect Phasor Project (EIPP) data and Section 5 draws the conclusion.

2. Description of real time WAMS data

On 1st June 2010 at 23:49 Hrs, an abnormal event of generation loss of 2000 MW took place at Rihand super thermal power station (STPS) I & II area while attending the stuck breaker issue of tie bay of 400 kV Rihand high voltage direct current (HVDC) and Singrauli STPS on its either side. Rihand STPS is located at Rihandnagar in Uttar Pradesh. It is one of the coal based power plants of NTPC and has an installed capacity of 2000 MW. Fig. 1 shows snapshot of Rihand STPS network connectivity. Rihand STPS switchyard of NTPC has one & half breaker scheme. As reported by NTPC, at Rihand STPS, the tie circuit breaker of 400 kV Singrauli-I & HVDC-2 bay dia went under lockout due to low air pressure in R- pole of the breaker. In order to attend the tie breaker under lockout, the main breakers of Singrauli-I & HVDC-II were manually opened at Rihand STPS end. Subsequently, 400 kV Rihand-Singrauli line-I (as in Fig. 2) was manually opened from Singrauli end. As reported by NTPC, a sharp dip in voltage (35 kV) was observed while opening the tie bay isolator (1189A), towards Singrauli-I. Immediately after this, both the units (1 & 2) of Rihand STPS stage-I tripped on pole slip protection and 400 kV Rihand-Singrauli-II tripped at Rihand STPS end on receipt of direct trip command from Singrauli end, and the 400 kV Bus-A at Rihand HVDC also became dead due to tripping of all the circuit breaker (CB) connected with this bus. The main breakers connected to HVDC Bus-A at Rihand HVDC also got opened due to the operation of local breaker back up (LBB) protection of tie breaker under lockout. During the incident, no large loss of load occurred except for a small number of events of load shedding due to under frequency relay (UFR) operation and a sharp dip in voltage (35 kV) was observed. After 7 s of tripping of Rihand stage-I, unit 3 of Rihand stage-II tripped due to forced draft (FD) fan off and tripping of unit 4 takes place due to lack of fuel supply. However, the HVDC Rihand-Dadri bipole & 400 kV Rihand-Allahabad D/C remained intact. At the time of disturbance, HVDC bipole was carrying 750 MW. Immediately after the tripping of Unit-III & Unit-IV the power flow in 400 kV Rihand-Allahabad - I & II lines reversed as power started flowing from Allahabad to Dadri vide Rihand-Dadri HVDC link. Phasor measurement units (PMU) which were installed at four stations namely Vindyachal, Dadri, Kanpur and Moga were used for analyzing the incident.

This paper presents processed PMU measured signals at four stations (Vindyachal, Dadri, Kanpur and Moga) for estimating the low frequency modes. The collected signals are basically the plots of the fault current plotted against a common time stamp by the
phasor measurement units and has a sampling frequency of 4 ms. The signals obtained from the PMUs consists of two zones namely, the ambient zone which is representative of the pre-disturbance zone and a decaying ring down zone of the signal which is representative of the post fault conditions prevailing in the power system network. The ringdown portions are taken for analysis in this paper. Figs. 2–5 shows the plot of collected signal at the four stations i.e. Vindyachal, Dadri, Kanpur and Moga respectively. The signal in Fig. 2–5 has ambient as well as ringdown portion. As the ringdown portion provides rich information of the low frequency oscillatory modes, so in this paper it is considered for further analysis. Figs. 6–9 shows the ringdown portions of Vindyachal, Dadri, Kanpur and Moga respectively. Pre-processing of the PMU collected signal is done by detrending. The statistical operation of removing trend from the signal is called detrending.

3. Signal processing based methods for estimating low frequency modes

3.1. Fourier Transform

The Fourier Transform breaks a function or a signal into a version which is characterized by sine and cosines [21] and provides a exclusive way of screening any function - as the summation of simple sinusoids. An algorithm to analyze the discrete Fourier Transform (DFT) is fast Fourier Transform (FFT) [22,23]. Conversion of time (or space) to frequency and vice versa is done by Fourier analysis and FFT quickly analyzes such transformations into a product of sparse (mostly zero) factors by factorizing the DFT matrix [21]. FFT re-expresses the discrete Fourier Transform (DFT) of an arbitrary composite size \( N = N_1N_2 \) in terms of smaller DFTs of sizes \( N_1 \) and \( N_2 \) recursively in order to minimize the estimation time to \( O(N \log N) \) [22]. The most used type of FFT is radix-2 decimation-in-time (DIT) [22]. A DFT of size \( N \) is divided into two interleaved DFTs (hence the name “radix-2”) of size \( N/2 \) by Radix-2 DIT with each recursive stage. Radix-2 DIT first computes the DFTs of the even-indexed inputs \( x_{2n} = \{x_0, x_2, \ldots, x_{N-2}\} \) and of the odd-indexed inputs \( x_{2n+1} = \{x_1, x_3, \ldots, x_{N-1}\} \) and then combines those two results to create the total DFT series [22]. This thought can then be worked out rigorously to minimize the total runtime to \( O(N \log N) \) [22]. Further the simplified form of DFT assumes that \( N \) is a power of two since the number of sample points \( N \) can generally be selected liberally by the application [22]. FFT is quick, in-sensitive to noise and easy to apply [23,24].

3.2. Prony method

Extension of Fourier analysis is Prony analysis which inspect signals to find frequency, damping coefficients and relative data within the signal [25]. Prony method use to give linear combination of exponentials from sampled data modeling and behaves like least squares linear prediction algorithm which is applied for AR (Autoregressive) and ARMA (Autoregressive moving average) parameter detection. Besides this, Prony analysis is a technique of fitting a linear group of exponential terms to a signal which is shown in (1) below.

\[
Y_k = \sum_{n=0}^{N-1} A_n e^{\sigma_n t} \cos(2\pi f_n t + \theta_n)
\]

where

\( A_n = \) Amplitude of nth mode  
\( \sigma_n = \) Damping factor of nth mode  
\( f_n = \) Frequency of the nth mode  
\( \theta_n = \) Phase angle of nth mode  
\( n = 1,2,3,\ldots \)

Each exponential term possessing distinct frequency is seen as a distinctive mode of the original signal \( y(t) \). The four terms of each mode as in (1) can be found from the state space presentation of an evenly sampled data record. The Prony analysis of the raw signals is made with the help of a prony toolbox which is a programmed graphic user interface which accepts the input as a raw signal and evaluates a prony approximation of the input signal. The mean square error involved with the approximation or in other words the precision of the technique is affected by varying the parameters of the toolbox which are known as the Model Order and the Residues. On proper selection of the values of model order and
the value of residues one can accurately find a prony approximate to the signal. Before the data is fed into the toolbox, a number of additional advantage is provided which includes removal of the mean i.e. the removal of the DC offset present in the signal, signal detrending and selection of the data range of the signal. The algorithm of the Prony analysis is done with the help of the prony toolbox in the present work which is algorithmically shown by a flowchart in Fig. 10. The Prony toolbox consists of four GUI, the details of which is [26]:-

3.2.1. Prepare prony data GUI
Main features of this GUI are:-
(i) Import data:- Importing data to the toolbox
(ii) Decimation:- It performs M-fold decimation
(iii) Data range selection:- It specifies a specific section of the decimated or undecimated signal.
(iv) Data preprocessing:- Preprocessing techniques available are:- removing mean and deterend (removing linear trends from the signal)

3.2.2. Perform prony analysis (PA) GUI
Main features of this GUI are:-
(i) Model order (O) selection:- N/3 < O < N/2. N = no of data samples.
(ii) Graphic mode: - This refers to the graphic display of the PA fit in time or frequency domain.
(iii) Total number of residues specified cannot exceed the modal order given.
(iv) Mode sorting criteria: GUI sorts the prony residues according to two criteria: amplitude and relative energy.

(v) Results: It displays amplitude, frequency, damping coefficient and relative energy of the signal.

(vi) Mean Squared error (MSE): MSE is the average of the squared error over the sampled data length. Evaluation of the performance of the prony fit is given by MSE.

3.2.3. Compare prony analysis (PA) sessions GUI

This GUI compares several PA sessions simultaneously. Main features of this GUI are:

(i) Compare Set Menu: For comparison of different sessions, user has to indicate the data to be compared by either importing the data from a CMP-file or from the workspace loading the data. Option is provided in this menu to open the CMP-file or to load the data from the workspace.

(ii) Sessions List: The GUI displays all the specified saved sessions in a list box. For each session it displays the data file name, data set, data preprocessing option, decimation option, decimation factor, data range option, model order, number of modes, mode sorting criteria and mode selection option.

(iii) Plots: The GUI plots the poles, squared error, energy and residues for the selected PA sessions.

3.2.4. Export data GUI

It exports data to MATLAB workspace as well as to a file.

3.3. The S-transform

S-transform is the modification of continuous wavelet transform and short-time Fourier Transform (STFT). S-transform has fixed modulation sinusoids with respect to the time axis as compared to wavelet transform. Due to this property of S-transform it can detect the scalable Gaussian window dilations and translations [27,28]. Local spectral phase properties cause the S-transform (ST) to be a time-frequency representation method [28]. Besides this, S-transform combines a time-frequency space frequency dependent resolution with absolutely referenced local phase information [28]. This tends to describe the phase in a local spectrum setting and gives a lot of beneficial characteristics. S-transform also exhibits a frequency invariant amplitude response as compared to the wavelet transform [28]. Continuous wavelet transform and S-transform are similar as both have progressive resolution but unlike the wavelet transform, S-transform retains absolutely referenced phase information and has a frequency invariant amplitude response [28]. Absolutely referenced phase information means that the information of the phase provided by the S-transform is the argument of the cosinusoid at zero time (which is similar to Fourier Transform) [28]. Local phase in S-transform is the peak in local spectral amplitude (indicating a quasi-monochromatic signal), as well as off peak, where the rate of change of the phase leads to a channel Instantaneous Frequency analysis [28]. Also, the S-transform detects the local power spectrum as well as local phase spectrum [28] and is related to the general complex valued time series. The S-transform is a representation of signal time and frequency and combines frequency dependent resolution with an ability to detect the real and imaginary spectra [29] simultaneously. The continuous S-transform of a function \( h(t) \) is defined as [28]:

![Fig. 6. Ringdown Portion of Dadri Station.](image1)

![Fig. 7. Ringdown Portion of Vindyachal Station.](image2)

![Fig. 8. Ringdown Portion of Moga Station.](image3)

![Fig. 9. Ringdown Portion of Kanpur Station.](image4)
3.4. Wigner Ville Distribution

The Wigner-Ville Distribution technique employs singular value decomposition (SVD) for spectral analysis of the signal to inspect the small frequency modes in the ringdown portion of the signal. This technique is based on segment extrapolation of a known autocorrelation function for the unknown lags [29]. The extrapolated autocorrelation function fulfilling the highest entropy criterion will be largely arbitrary one reliable with the known segment of the autocorrelation lags [29]. Equivalently, the detected spectrum is the smoothest one of all the spectra which do not clash with the identified segment of the autocorrelation lags corresponding to the kernel. The direct application of this method is subject to several well-known difficulties [29] which are:

- The choice of the order of the AR model;
- Since the method depends on a least-squares fitting of the kernel to the AR model, it is noise susceptible which is preservative with the signal.

The method improves robustness to the noise and makes it robust to the order of the AR model. This is obtained by singular value decomposition (SVD) of the kernel data matrix to calculate a small rank principle eigenvector approximation to the inverse covariance matrix in the AR modelling technique [29]. WVD applies the singular value decomposition to form the principle eigenvector approximation to the inverse covariance matrix. It can determine the correct onset of changes in natural frequency i.e. all temporal resolution is contained in the phase of the transform [29]. The Wigner-Ville Distribution (WVD) function is expressed as [29]:

\[
\text{WVD}(\tau, \omega) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2})^* x(t - \frac{\tau}{2}) e^{-j\omega \tau} d\tau
\]  

(3)

where \( t \) is a time variable, \( \omega \) is a variable of frequency, \( \tau \) is the lag and \( * \) denotes complex conjugate.

3.5. Estimation of low frequency modes by rotational invariance technique (ESPRIT)

The ESPRIT gives the property of rotational shift invariance of the signals for the development of an auto-correlation matrix and transforms it into a generalized eigen value problem which is solved by ESPRIT [30]. The correlation matrix is developed by ESPRIT which exactly uses the data samples compared to Prony-based methods. Therefore ESPRIT is computationally more expensive but have much high immunity towards noise [30]. The assumed noise in the ESPRIT model is additive white noise. As PMU uses an anti-aliasing filter before sampling, so the acquired power signals consisted of small high immunity towards noise [30]. The noise model of power system in the projected method has exponentially damped sinusoids with additive white noise [30]. For a given data mass from the considered signal model of the signal [30] is given as:-

\[
y(n) = s(n) + w(n) = \sum_{k=1}^{K} a_k b_k e^{j\omega_k n} \cos(\omega_k n + \phi_k)
\]

(4)

where \( s(n) \) is the signal, \( w(n) \) the zero-mean noise, \( a_k \) is the amplitude, \( \omega_k \) is the angular frequency, \( \phi_k \) is the initial phase, \( K \) is the number of sinusoids and \( b_k \) the attenuation factor. Eq. (3) can be described in the complex exponential form as:-

\[
y(n) = s(n) + w(n) = \sum_{j=1}^{M} x_j e^{j\omega_j n} + a_n
\]

(5)

\[
S(\tau, f) = \sum_{\tau=1}^{\infty} h(t) - |f_i|^2 e^{-j2\pi f_i \tau} d\tau
\]

A “voice” \( S(\tau, f_0) \) is a one dimensional function of time for a constant frequency \( f_0 \) which shows the variation of amplitude and phase for this particular frequency over time. A “local spectrum” \( S(t_0, f) \) is a one dimension frequency function for a constant time \( t_0 \) [28].
where \( \alpha_j = \frac{f_j}{2}, \beta_j = b_j + i\omega_j \), \( M = 2K \).

A modified ESPRIT based method, called TLS-ESPRIT (Total least square-estimation of system parameter by rotational invariance technique) implements the property of first and second rotational shift invariance of the signal to minimize the coloured Gaussian noise cause which is formed due to filtering of the signal containing an additive Gaussian noise [30]. Further minimization of coloured noise is done by signal transformation into the eigen vector. Besides this, ESPRIT minimizes the storage and computational cost [30]. The algorithm of the discussed method is shown in Fig. 11. A block of \( N_1 \) which has majority of recent samples collected from the PDC is implemented by the projected method, where \( N_1 \) is approximately taken to be the ratio of the phasor data rate of the PMU and the lower limit of the frequency of the estimator which are then passed through a down-sampler to find the autocorrelation matrix [30].

### 3.6. Hilbert-Huang Transform (HHT)

While Wigner-Ville Distribution is applicable for non-stationary linear data, in most real systems, the data are generally nonlinear and non-stationary which can be analyzed by empirically based data-analysis method called Hilbert–Huang transform [31]. Hilbert-Huang Transform is adaptive in nature, so that it can make significant time-frequency representations of data from nonlinear and non-stationary processes [32]. The HHT consists of two parts: empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). This new method is direct, spontaneous and adaptive, with a posteriori-defined basis, from the decomposition method, based on and derived from the data. The decomposition is based on the simple assumption that any data consists of different simple intrinsic modes of oscillations. Each intrinsic mode, linear or nonlinear, represents a simple oscillation, which will have the same number of extrema and zero-crossings. Furthermore, the oscillation will also be symmetric with respect to the “local mean.” At any given time, the data may have many different coexisting modes of oscillation, one superimposing on the others. The result is the final complicated data. Each of these oscillatory modes is represented by an intrinsic mode function (IMF) with the following definition:

\[
x(t) = R \left\{ \sum_{j=1}^{n} a_j \exp \left[ i \int w_j(t) dt \right] \right\}
\]

Eq. (7) gives both the amplitude and frequency of each component as functions of time. The same data expanded in a Fourier representation would be [31]:

\[
x(t) = R \left\{ \sum_{j=1}^{n} a_j e^{i\omega_j t} \right\}
\]

The contrast between Eq. (6) and Eq. (7) is clear that the IMF represents a generalized Fourier expansion. The variable amplitude and the instantaneous frequency have not only greatly improved the efficiency of the expansion, but also enabled the expansion to accommodate non-linear and non-stationary data [31]. With the IMF expansion, the amplitude and the frequency modulations are also clearly separated [31,32]. Thus, the restriction of the constant amplitude and fixed frequency of the Fourier expansion has been overcome, with a variable amplitude and frequency representation [31,32]. This frequency-time distribution of the amplitude is designated as the “Hilbert amplitude spectrum” \( H(\omega, t) \), or simply “Hilbert spectrum.”

### 3.7. Matrix Pencil Method (MPM)

Matrix Pencil is a popular method to extract parameters from exponentially damped/undamped signal which is based on the property of the underlying signal [33]. The details of the algorithm of the method is given in [33,34]. MPM is computationally efficient and less sensitive to noise [33].

### 4. Simulation results

#### 4.1. Estimation of low frequency modes by FFT

The signals shown in Figs. 6–9 is analyzed with the help of FFT function in the MATLAB which yields the dominant low frequency modes of the oscillations during the fault conditions. In FFT analysis, Gaussian window has been taken whose width is adjusted according to the signal by trial and error. During FFT of the signal, detrending is done. Figs. 12–15 shows the magnified portion of the Fourier Transform plot obtained, which shows the dominant low frequency modes in the unit of hertz (Hz) and the amplitude level associated with those frequencies in the units of ampere (A). The entire information is provided with numerous data tips in the magnified portions of the plot to facilitate quick reference of the results obtained. The FFT plot generally extends up to an infinite distance along the X axis. Hence, for the sake of brevity the portion of the curve has been focused which provided the information on the dominant modes of frequencies. Similarly on the Y axis, the entire
data plot has not been shown because of the aforementioned reasons. The plot of the Fourier Transform has been shown in Figs. 12–15 which shows the plot of the frequency points of the signal and the respective amplitude levels associated with those spectral frequencies in the units of ampere and hertz. Dominant low frequency modes of Dadri, Vindyachal, Kanpur and Moga obtained by FFT are set in Table 1. It can be visualized from the Table 1 that three frequencies (0.26, 0.52 and 1.73) are common among all the four stations.

4.2. Estimation of low frequency modes by prony analysis

The signals obtained from the four PMUs are analyzed using a Prony Tool Box Graphic User Interface, which is an algorithm based user interface program used in order to find out the certain signal parameters like frequency, amplitude, damping coefficient and relative energies respectively. This toolbox accurately plots the prony approximate of the applied raw signal fed into it after certain preprocessing techniques like mean removal, signal detrending etc. It hence plots and reports the mean square error produced due to such approximation and plots certain additional useful plots of mean square error or pole-zero diagram. The prony approximate of the raw signal collected from the PMU and its mean square plot at the Vindyachal, Dadri, Kanpur and Moga station are shown in Figs. 16a–16d respectively. It can be observed from Figs. 16a–16d that the mean square error for all the four stations are very less. Table 2 gives the distribution of amplitude and frequency obtained from the prony analysis. It can be observed from Table 2 that Vindyachal, Dadri, Moga and Kanpur has five frequencies (0.65, 0.26, 0.52, 0.78, 1.73) common among them. Also it can be noticed from Table 1 and Table 2 that four low frequency modes are common for the four stations (Vindyachal, Dadri, Moga and Kanpur) by FFT and PA which is given in Table 3.

4.3. Estimation of low frequency modes by S-Transform

The S transform of the raw signal collected from the four PMUs at Vindyachal, Dadri, Kanpur and Moga as shown in Figs. 2–5 was performed in order to draw the contour plot of their frequencies which is shown in Figs. 17–20. The length of the time series is 200 points and the sampling rate is 40 ms. The dominant low frequency modes of the disturbances were obtained from region which had more violet colourations representing the ringdown portion of the signal as compared to the region which were more lighter in colouration representing the ambient portion of the signal. Some of the dominant frequencies has been shown in the contour plot for clarity. In the contour diagram, x-axis denotes the
time, y-axis indicates low frequency modes and z-axis denotes the amplitude. The low frequency dominant modes of the four station (Vindyachal, Dadri, Kanpur and Moga) is shown in Table 4. In Table 4, other detail parameters (amplitude and time) of the signal is also given. It can be analyzed from Table 4 that there are approximately four common dominant low frequency modes (0.7, 0.5, 1.7, 0.2) among the four station (Vindyachal, Dadri, Kanpur and Moga). The common frequencies by FFT, PA and S-T is given in Table 5. It can be found from Table 5 that the common approximate frequency (up to one decimal) is four.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Amplitude</td>
<td>Frequency</td>
<td>Amplitude</td>
</tr>
<tr>
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<td>0.09</td>
<td>0.086</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>0.006</td>
<td>0.003</td>
<td>0.26</td>
</tr>
<tr>
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<td>0.69</td>
<td>0.015</td>
<td>0.004</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
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<td>0.004</td>
<td>0.0007</td>
<td>0.52</td>
</tr>
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<td>6</td>
<td>1.21</td>
<td>0.007</td>
<td>0.001</td>
<td>1.73</td>
</tr>
<tr>
<td>7</td>
<td>1.73</td>
<td>0.001</td>
<td>1.73</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Fig. 16a. Prony analysis of Vindyachal station (i) raw signal collected from the PMU, (ii) mean square error plot.

Fig. 16b. Prony analysis of Dadri station (i) raw signal collected from the PMU, (ii) mean square error plot.

Fig. 16c. Prony analysis of Kanpur station (i) raw signal collected from the PMU, (ii) mean square error plot.

Fig. 16d. Prony analysis of Moga station (i) raw signal collected from the PMU, (ii) mean square error plot.
4.4. Estimation of low frequency modes by Wigner-Ville Distribution

As the aspire of the study was to find the possibility of improving the frequency resolution of the WVD without losing its valuable properties such as estimation of instantaneous frequency, so main concern is in detection of the leading oscillating modes of the raw signals acquired from the four PMUs at Dadri, Vindyachal,

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal Frequency</th>
<th>Amplitude</th>
<th>Dadri Frequency</th>
<th>Amplitude</th>
<th>Moga Frequency</th>
<th>Amplitude</th>
<th>Kanpur Frequency</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.011</td>
<td>0.26</td>
<td>0.011</td>
<td>0.26</td>
<td>0.015</td>
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</tr>
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<td>0.35</td>
<td>0.015</td>
<td>0.30</td>
<td>0.013</td>
<td>0.34</td>
<td>0.018</td>
<td>0.31</td>
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<td>0.008</td>
<td>0.52</td>
<td>0.007</td>
<td>0.52</td>
<td>0.010</td>
<td>0.52</td>
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<td>0.65</td>
<td>0.018</td>
<td>0.65</td>
<td>0.016</td>
<td>0.65</td>
<td>0.019</td>
<td>0.65</td>
<td>0.019</td>
</tr>
<tr>
<td>5</td>
<td>0.78</td>
<td>0.007</td>
<td>0.78</td>
<td>0.004</td>
<td>0.78</td>
<td>0.005</td>
<td>0.78</td>
<td>0.013</td>
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<tr>
<td>6</td>
<td>1.27</td>
<td>0.0010</td>
<td>1.11</td>
<td>0.009</td>
<td>1.05</td>
<td>0.013</td>
<td>1.21</td>
<td>0.015</td>
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<td>1.73</td>
<td>0.003</td>
<td>1.73</td>
<td>0.002</td>
<td>1.73</td>
<td>0.003</td>
<td>1.73</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Table 2
Parameter estimation by prony analysis.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.26</td>
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<td>2</td>
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<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>4</td>
<td>1.73</td>
<td>1.73</td>
<td>1.73</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Table 3
Common frequency modes by FFT and PA.

Fig. 17. Contour plot of S-Transform of Vindyachal Station.

Fig. 18. Contour plot of S-Transform of Dadri Station.

Fig. 19. Contour plot of S-Transform of Kanpur Station.

Fig. 20. Contour plot of S-Transform of Moga Station.
Moga, and Kanpur, the sequential analysis is carried out on a continuous basis. The comparison of oscillating frequencies with different locations is done in WVD. Figs. 21–24 shows the low frequency plot for the four stations (Vindyachal, Dadri, Kanpur and Moga) by WVD. Some of the dominant low frequencies as revealed in Figs. 21–24 is given in the Table 6. It can be analyzed from Table 6 that four frequencies are common among the four sta-

### Table 4
Parameter estimation by S-Transform.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Amplitude</td>
<td>Time</td>
<td>Frequency</td>
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<tr>
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<td>0.22</td>
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<td>0.26</td>
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<tr>
<td>2</td>
<td>0.55</td>
<td>0.008</td>
<td>98.6</td>
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<td>0.66</td>
<td>0.010</td>
<td>101.6</td>
<td>0.72</td>
</tr>
<tr>
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<td>0.77</td>
<td>0.009</td>
<td>100.3</td>
<td>0.79</td>
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<td>1.22</td>
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<td>99.2</td>
<td>0.85</td>
</tr>
<tr>
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<td>1.33</td>
<td>0.013</td>
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<tr>
<td>7</td>
<td>1.77</td>
<td>0.0071</td>
<td>99</td>
<td>1.71</td>
</tr>
</tbody>
</table>

### Table 5
Common frequency modes by FFT, PA and S-Transform.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
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</thead>
<tbody>
<tr>
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<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
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<tr>
<td>4</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Fig. 21. Low frequency plot for Vindyachal station using Wigner-Ville Distribution.

Fig. 22. Low frequency plot for Dadri station using Wigner-Ville Distribution.

Fig. 23. Low frequency plot for Kanpur station using Wigner-Ville Distribution.

Fig. 24. Low frequency plot for Moga station using Wigner-Ville Distribution.

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frequencies (Vindyachal, Dadri, Kanpur and Moga). Further it is also noticed that 02 frequencies are common for the four stations by FFT, PA, S-Transform and WVD which is given in Table 7.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>1.01</td>
</tr>
<tr>
<td>4</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.24</td>
</tr>
<tr>
<td>5</td>
<td>1.74</td>
<td>1.74</td>
<td>1.73</td>
<td>1.77</td>
</tr>
<tr>
<td>6</td>
<td>2.24</td>
<td>2.49</td>
<td>2.48</td>
<td>2.52</td>
</tr>
<tr>
<td>7</td>
<td>2.49</td>
<td>2.49</td>
<td>2.48</td>
<td>2.52</td>
</tr>
</tbody>
</table>

Table 7
Common frequency modes by FFT, PA, S-TRANSFORM and WVD.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</tr>
</tbody>
</table>

Fig. 25. Low frequency plot for Vindyachal station using ESPRIT.

Fig. 26. Low frequency plot for Dadri station using ESPRIT.

Fig. 27. Low frequency plot for Kanpur station using ESPRIT.

Fig. 28. Low frequency plot for Moga station using ESPRIT.

Fig. 29. Low frequency plot for Vindyachal station using HHT.

Fig. 30. Low frequency plot for Dadri station using HHT.

Fig. 31. Low frequency plot for Kanpur station using HHT.
4.5. Estimation of low frequency modes by ESPRIT

Some of the dominant low frequency modes of the four stations (Vindyachal, Dadri, Kanpur and Moga) are shown in Figs. 25–28 and tabulated in Table 8. It can be observed from Figs. 25–28 and Table 8 that 0.26 and 0.52 are the common frequencies among the four stations. Table 9 shows that 0.2 and 0.5 are the two common frequencies of the ESPRIT with methods discussed earlier in this paper. The parameters of ESPRIT is given in Appendix 'A'.

4.6. Estimation of low frequency modes by HHT

Some of the dominant low frequency modes of the four stations (Vindyachal, Dadri, Kanpur and Moga) are shown in Figs. 29–32 and tabulated in Table 10. It can be observed from Figs. 29–32...

Table 10
Dominant low frequency modes by HHT.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.27</td>
<td>0.22</td>
<td>0.27</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>0.52</td>
<td>0.25</td>
<td>0.54</td>
</tr>
<tr>
<td>3</td>
<td>0.69</td>
<td>0.75</td>
<td>0.54</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>0.78</td>
<td>1.77</td>
<td>1.28</td>
<td>1.76</td>
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</table>

Table 11
Common frequency modes by FFT, PA, S-TRANSFORM, WVD, ESPRIT and HHT.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 12
Dominant low frequency modes by MPM.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.26</td>
<td>0.27</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>0.53</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td>3</td>
<td>0.70</td>
<td>0.93</td>
<td>0.73</td>
<td>0.96</td>
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<tr>
<td>4</td>
<td>1.75</td>
<td>1.11</td>
<td>0.94</td>
<td>1.71</td>
</tr>
</tbody>
</table>

Table 13
Common frequency modes by FFT, PA, S-TRANSFORM, WVD, ESPRIT, HHT and MPM.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Vindyachal</th>
<th>Dadri</th>
<th>Moga</th>
<th>Kanpur</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</tbody>
</table>
and Table 10 that approximately 0.26 and 0.52 are the two common frequencies among the four stations. Table 11 shows that 0.2 and 0.5 are the two common frequencies of the HHT with methods discussed earlier in this paper.

4.7. Estimation of low frequency modes by Matrix Pencil Method

For estimating the low frequency modes, singular value decomposition in combination with Matrix Pencil Method is used. The algorithm of the projected method is taken from [35]. Some of the dominant low frequency modes of the four stations (Vindyachal, Dadri, Kanpur and Moga) are shown in Figs. 33–36 and tabulated in Table 12. It can be observed from Figs. 33–36 and Table 12 that approximately 0.26 and 0.52 are the two common frequencies among the four stations. Table 13 shows that 0.2 and 0.5 are the two common frequencies of the Matrix Pencil Method with methods discussed earlier in this paper. The parameters of Matrix Pencil Method is given in Appendix ‘A’.
4.8. Comparison of the above projected methods for estimation of low frequency modes

It can be observed from Table 1 that FFT gives 03 exactly common frequencies (0.26, 0.52, 1.73) among the four stations (Vindyachal, Dadri, Kanpur and Moga). Prony analysis gives more number of exactly common frequencies i.e. 05 (0.26, 0.52, 0.65, 0.78, 1.73) among the four stations (Vindyachal, Dadri, Kanpur and Moga) as shown in Table 2. From Table 4, it can be noticed that S-Transform gives 04 approximate frequencies which is common if considered up to one decimal (0.2, 0.5, 0.7, 1.7) among the four stations (Vindyachal, Dadri, Kanpur and Moga). ESPRIT gives two exactly common frequencies (0.26, 0.52) among the four stations (Vindyachal, Dadri, Kanpur and Moga) as observed from Table 8. From Table 10 and Table 12 it can be noticed that HHT and MPM gives two approximate frequencies which is common if considered up to one decimal (0.2, 0.5, 0.7, 1.7) among the four stations (Vindyachal, Dadri, Kanpur and Moga). So, it can be concluded that FFT, PA and ESPRIT estimates the low frequency oscillatory modes much more accurately than the other methods discussed. However due to large sensitivity towards noise of PA, it gives spurious modes and computational burden associated with it is more. FFT and ESPRIT has emerged as a powerful and quick method and so they are recommended for estimating the oscillatory low frequency modes from ringdown signals.

4.9. Estimation of low frequency oscillating modes of Eastern Interconnect Phasor Project (EIPP), U.S

The real-time WAMS data for low frequency mode estimation is taken from EIPP, U.S for an event on 20th July 2005 [36,37]. The raw signal is shown in Fig. 37. Estimation of low frequency oscillatory dominant modes is done by FFT, PA, S-Transform, WVD, ESPRIT, HHT and MPM which is shown in Figs. 38–44 respectively. The low frequency modes are tabulated in Table 14. It can be observed from Table 14 that exactly one frequency i.e. 0.51 Hz is common among all the methods. Also noticed from Table 14 that 0.2 Hz is also common among all methods if one decimal place is taken. To evaluate the performance of the discussed signal processing based methods, results of low frequency oscillatory modes are verified from [36]. It can be observed from [36] that 0.5 and 0.2 are the oscillatory modes which verifies the simulation results and the performance accuracy of the discussed methods.

5. Conclusion

This paper discusses about a practical case study to estimate the dominant low frequency oscillatory modes of the signals that led to a generation loss. Seven signal processing based methods were used to pinpoint the dominant low frequency modes of oscillations in the ring down portion of the signal which define the small signal stability of the system under consideration. These seven signal processing based techniques used were Prony Analysis, Fast Fourier Transform, Stockwell Transform, Wigner Ville Distribution, Estimation of Signal Parameters by Rotational Invariance Technique, Hilbert-Huang Transform and Matrix Pencil Method. These meth-

<table>
<thead>
<tr>
<th>Mode</th>
<th>FFT</th>
<th>PA</th>
<th>ST</th>
<th>WVD</th>
<th>ESPRIT</th>
<th>HHT</th>
<th>MPM</th>
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</thead>
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<td>0.23</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>2</td>
<td>0.51</td>
<td>0.51</td>
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<td>0.51</td>
</tr>
<tr>
<td>3</td>
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<td>0.95</td>
<td>0.99</td>
<td>0.96</td>
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<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>0.81</td>
<td>1.25</td>
<td>1.24</td>
</tr>
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</table>

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ods, as expected, have their own inherent advantages and disadvantages associated with their algorithms in finding the low frequency modes. Nevertheless, the frequencies obtained from these analysis are more or less uniform and coherent, hence, suggesting the efficacy and accuracy of the signal processing methods used in the study. The Prony analysis of the signals yielded the low frequency modes in the ring down portions of the signal but is affected by the presence of external noise and fluctuations as a result of which gives spurious frequencies and difficult to estimate actual frequencies present in the signal. After this we have the Fast Fourier Transform of the raw signals which yielded the dominant low frequency modes with the corresponding amplitudes. In this method we had the convenience of adjusting the window function to fit the bandwidth of the raw signal to obtain a better resolution among all other methods. The S transform of the signals provided a contour plot which helped in easily finding the zone of disturbance. Hence, the Stockwell transformation of signals helped in pinpointing the low frequency modes of oscillations associated with the zone of disturbances more accurately. The Wigner distribution helped in obtaining the contour plot of the signals, hence, it made it easy in finding the low frequency modes of oscillation. The values of dominant low frequency modes of oscillations obtained by the ESPRIT technique is very nearly corresponds to the values of the frequency obtained from that of the earlier discussed methods. It is observed that the discussed ESPRIT method in this manuscript performs much better in terms of the variance and has the negligible bias in the presence of the coloured Gaussian noise, which is mainly generated due to the presence of the low pass Butterworth filter. IHT and MPM estimates the significant low frequency modes present in the signals and the results are in accordance with the results obtained from previous discussed signal processing techniques.

Further from the simulation results it was found after comparison of all dominant frequencies of the four station (Vindyaachal, Dadri, Kanpur and Moga) by the seven discussed signal processing based technique (Prony Analysis, Fast Fourier Transform, Stockwell Transform, Wigner Ville Distribution, Estimation of Signal Parameters by Rotational Invariance Technique, Hilbert-Huang Transform and Matrix Pencil Method) that two dominant frequencies are common i.e. 0.2 and 0.5 Hz. To evaluate the performance of the discussed seven techniques (Prony Analysis, Fast Fourier Transform, Stockwell Transform, Wigner Ville Distribution, Estimation of Signal Parameters by Rotational Invariance Technique, Hilbert-Huang Transform and Matrix Pencil Method) in the manuscript, investigations were carried out to estimate low frequency oscillatory modes of EIPP data and it was observed from the simulation results that there are two oscillatory mode frequency i.e. 0.51 Hz and 0.2 Hz. The simulation results were checked with the reference [36].

Acknowledgements

The author would like to thank Science and Engineering Research Board (SERB), Department of Science and Technology (DST) for funding the work under Young Scientist Research Scheme.

Appendix A

A.1. Parameters of the Fast Fourier Transform

Gaussian Window Factor ($\sigma$):

(a) For Vindyaachal Station: 0.2
(b) For Dadri Station: 0.3
(c) For Kanpur Station: 0.2
(d) For Moga Station: 0.5

A.2. Parameters of the Prony analysis

(a) Model Order: 07
(b) Residues: 07

A.3. Parameters of the S-transform

(a) Width of the localizing Gaussian window: 3
(b) Sampling rate: 0.04
(c) Frequency sampling rate: 1

A.4. Parameters of esprit

Factor = 5

A.5. Parameters of MPM

Real Damped Sine Components = 300

References


