Fully coupled EHL model for simulation of finite length line cam-roller follower contacts

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ABSTRACT

This paper presents a fully coupled multiphysics model developed for numerical simulation of the elastohydrodynamically lubricated contact between a cam and roller, typically found in cam mechanisms controlling fuel injection in an internal combustion engine. Special attention was focused on enabling systematic analyses of effects associated with, roller crowning and edge geometries, lubricant rheology as well as typical cam mechanism operating conditions. The interaction between the cam and roller follower result in high generated pressure and narrow film thickness that increase the risk of wear and fatigue. Furthermore, this article highlights the variation of pressure and film thickness under tilting conditions. The model was also validated against some particular model problems, found in the literature.

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1. Introduction

Cam mechanisms represent a major class of system that interacts under elastohydrodynamic lubrication (EHL) conditions. The counter-formal contact between the cam and follower is usually subjected to moderate-to-high loads which leads to localized elastic deformation.

Moreover, the interaction between the cam and roller follower changes with operating conditions, i.e. load, velocity and radii of curvature. The high load, the variation in speed and in contact size (radii of curvature changes with cam rotation) bring up some major issues in terms of friction and wear. In a typical vehicle, the contact between the cam and the roller follower accounts for 6–10% of all the frictional power loss, see Andersson [1]. In addition, wear and fatigue are caused by the high contact pressures generated in these concentrated contacts. Hardening and/or coating of the components are therefore usually required, see Teodorescu and Rahnejat [2]. As the result of the above issues, as well as high shear rate and increased flash temperature in the contact, the film thickness is typically extremely thin, leading to surface interactions and thus wear, see Dyson and Naylor [3].

Cam mechanisms have a wide range of industrial applications. A great variety of roller designs exist because optimizing the roller’s geometry can greatly reduce surface stresses and damages. Cam-follower contact edge stress concentration occurs due to discontinuity in profile at the edges, so they are usually slightly profiled in order to reduce these pressures, see the works of Kushwaha and Rahnejat in [4,5], and Teodorescu et al. [6]. Edge loading also reduces the life of roller bearings, see Lundberg and Palmgren [7].

The formation of an EHL film effectively prevent the asperities to be in close contact. Fundamental understanding of film formation, in this type of conjunctions with continuously changing contact conditions, is required in order to prevent wear. Gohar and Cameron [8] compared experimental results obtained using ball-on-disc and roller-on-disc test devices, and they observed that the minimum film thickness occurred at the sides of the contact zone (boundary of Hertzian contact). This finding was subsequently confirmed experimentally by Wymer and Cameron in [9]. Also, some numerical simulation conducted to show the behavior of film thickness in EHL finite line conjunctions. A range of rollers with different crownings, logarithmic profiles, chamfering, and rounded edges were numerically simulated and the influence of the edge properties on the pressure and film thickness thoroughly discussed, i.e. see Johns and Gohar [10], Mostofi and Gohar [11], Kuroda and Arai [12], Kushwaha and Rahnejat [5] and Najjari and Guibault [13].

In this work, both steady state and time dependent simulations of cam-roller follower systems are carried out in order to predict the variation in film thickness and contact pressure distribution during an engine cycle. The model is based on the one presented by Habchi et al. [14] and modified to consider finite line contacts. The steady state solutions provided useful information regarding the effects of contact geometry between a finite length roller with a logarithmic profile and rounded edges. Steady state simulations were also performed to investigate the effect of tilting, for a few
different roller geometries and under some different operating conditions, on lubricant performance. A time dependent analysis was conducted in order to capture the squeeze effect and other transients arising due to variation in the operating conditions during the engine cycle. The elastic deformation of the cam and roller is obtained using a modified system of equations for classical linear elasticity. The present approach makes use of the stabilized Galerkin least squares finite element method to deal with the severe instabilities encountered under conditions involving high loads. This is the main contribution over the published reference by [4]. The results of the steady state analyses conducted by means of the present model are compared to those obtained using the numerical approach reported by Kweh [15] and Nijenhanning [16]. Moreover, to be used as a benchmark, time dependent simulation of 3D-line contact is performed and the results compared with results for a crowned roller.

2. Elastohydrodynamic governing equations

The pressure distribution inside the lubricant film is obtained by solving the time-dependent Reynolds equation governing Newtonian fluid flow in thin interfaces between surfaces in relative motion (This approach is also used to conduct the steady state simulations.). It is the unidirectional pure rolling case that is considered and the velocities (parallel to the x-axis) are denoted $u_1$ and $u_2$, respectively. It is also assumed that the inlet of the contact is fully flooded and that the influence of surface roughness is small enough to be disregarded (idealized situation). Moreover, the analysis does not include coated and/or hardened surfaces.

The following set of scaling parameters are defined and applied to transform Reynolds equation in dimensionless form, note that the effects of side leakage is ignored in this analysis.

$$ X = \frac{x}{a}, \quad Y = \frac{y}{L}, \quad Z = \frac{z}{a}, \quad P = \frac{p}{P_h}, \quad H = \frac{h R_e}{a^2}, $$

$$ \bar{\eta} = \frac{\eta}{\eta_r}, \quad \bar{\rho} = \frac{\rho}{\rho_r}, \quad \bar{\theta} = \frac{\theta}{\theta_r}, \quad \bar{U_e} = \frac{U_e}{u_r}, \quad \bar{R_i} = \frac{R_i}{R_e}, $$

$$ T = \frac{t u_r}{a}, $$

where $\eta_r$ and $\rho_r$ represents the viscosity and density at the reference temperature, respectively and $a$, $P_h$ and $u_r$ represents the Hertzian contact radius, Hertzian maximum pressure and entrainment velocity, respectively. For line contacts, these parameters are defined as follows:

$$ a = \frac{8 F R_e}{\pi E L}, \quad P_h = \frac{2 F}{\pi a L}. $$

Hence, the dimensionless time dependent Reynolds equation can be represented as:

$$ \frac{\partial}{\partial X} \left( \frac{\epsilon}{\xi} \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{\epsilon}{\xi} \frac{\partial P}{\partial Y} \right) = \frac{\partial}{\partial T} \left( \rho \frac{\partial H}{\partial X} \right) + \frac{\partial}{\partial T} \left( \rho \frac{\partial H}{\partial T} \right), $$

where

$$ \epsilon = \frac{\eta H^2}{\bar{\eta}}, \quad \lambda = \frac{12 u_1 \eta R_e^2}{P_h a^2}, \quad \phi = \frac{a}{L}. $$

2.1. Lubricant properties

The relationship between density and pressure was taken to be that proposed by Dowson and Higginson [17]. This expression accounts for the compressibility of the lubricant to a certain extend. The main reason for not using a free volume EOS is that the quantitative data for compressibility of the lubricant was not available.

$$ \rho = \rho_0 \left[ 1 + \frac{0.6 \times 10^{-3} p}{1 + 1.7 \times 10^{-3} p} \right]. $$

Under the elastohydrodynamic lubrication regime, the viscosity of...
the lubricant increases by several orders of magnitude. To describe the dependence of lubricant viscosity on pressure, the modified WLF equation is employed. According to [18] the modified WLF reads as follows:

\[
q = \eta_0 \times 10^{\left( T_{gr}(p) - T_0 \right) / \Delta T}
\]

\[
T_g(p) = T_g(0) + A_1 \ln \left( 1 + A_2 p \right),
\]

where \( A_1, A_2, B_1, B_2, C_1 \) and \( C_2 \) are characteristic parameters for the lubricant and \( \eta_0 \) represents the viscosity at the glass transition temperature \( T_g \). The function \( T_g(p) \) represents the variation of the glass transition temperature with respect to pressure based on experimental data while \( F(p) \) represents the variation of the thermal expansion coefficient with pressure, see e.g. Habchi et al. [14].

2.2. Film thickness

The key variables in the EHL film thickness equation are usually the mutual separation of the two contact surfaces, \( h_0 \), geometry of the contact, \( g \), and the elastic deformation of the two surfaces, \( w \). Cylindrical rollers typically have profiled edges with crowning, chamfering, logarithmic, and corners to reduce the stress concentration and establish a smooth pressure distribution this must also be accounted for in the film thickness equation. In dimensionless form, the film thickness equation reads

\[
H(X, Y, T) = H_0(T) + G(X, Y, T) + W(X, Y, T),
\]

where the undeformed separation \( H_0(T) \) is determined by a force balance equation, \( G(X, Y, T) \) corresponding to the dimensionless contact geometry and \( W(X, Y, T) \) corresponding to the dimensionless elastic deformation, see Eq. (9).

A great diversity of roller shapes exists due to the diverse applications of rollers in machines. Rollers with axial crowning as shown in Fig. 1 are quite common; the geometry of such rollers is described by the following equations:

\[
g(x, y) = R_r - \sqrt{d^2 - x^2}, \quad d = R_r - R_c + \sqrt{R_c^2 - y^2},
\]

where \( R_r \) and \( R_c \) correspond to the roller and the crowning radius, respectively.

The elastic deformation of the cam and roller follower is estimated by applying a three-dimensional linear elasticity equation to the contacting structure. The following dimensionless parameters are used to rewrite the linear elasticity equation in dimensionless form.

\[
U = uR_r/a^2, \quad V = vR_r/a^2, \quad W = wR_r/a^2,
\]

where \( U, V, W \) correspond to the dimensionless deformation in the \( X, Y, Z \) direction, respectively. The system of equations for elastic deformation is thus as shown below.

\[
\frac{\partial}{\partial X} \left[ \left( \lambda_1 + 2\mu_1 \right) \frac{\partial U}{\partial X} + \nu_1 \frac{\partial V}{\partial Y} + \lambda_1 \frac{\partial W}{\partial Z} \right] + \phi \frac{\partial}{\partial Y} \left[ \mu_1 \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \right] + \frac{\partial}{\partial Z} \left[ \mu_1 \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) \right] = 0,
\]

\[
\frac{\partial}{\partial Y} \left[ \nu_1 \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \right] + \phi \frac{\partial}{\partial Z} \left[ \mu_1 \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial Y} \right) \right] + \frac{\partial}{\partial X} \left[ \lambda_1 \frac{\partial U}{\partial X} + \nu_1 \frac{\partial V}{\partial X} + \lambda_1 \frac{\partial W}{\partial Y} \right] = 0.
\]

By solving the above equations the deformation \( W(X, Y, T) \) can be obtained. For more details see the work of Habchi [19]. Note that Eq. (5)) in the reference [19] is modified by applying the \( \phi \) parameter. Both \( \lambda_1 \) and \( \mu_1 \) are the Lamé parameters

\[
\lambda_1 = \frac{\nu_{c,eq} E_{c,eq}}{(1 + \nu_{c,eq})(1 - 2\nu_{c,eq})}, \quad \mu_1 = \frac{E_{c,eq}}{2(1 + \nu_{c,eq})},
\]

in which \( \nu_{c,eq} \) and \( E_{c,eq} \) represents the equivalent poisson ratio and equivalent elastic modules for the contacting bodies, respectively. The equivalent elastic modulus is used in these equations for the

![Fig. 1. Schematic view of a crowned roller. Roller radius, \( R_r \) and crowning radius, \( R_c \).](image-url)
sake of simplicity. This means that instead of solving the equations for two domains, only one domain is needed to calculate the total elastic deformation of the mating surfaces. The following relation is used to define the equivalent elastic modulus:

\[
E_{eq} = \frac{E_1E_2(1 + \nu_2)^2 + E_2E_1(1 + \nu_1)^2}{E_1(1 + \nu_2) + E_2(1 + \nu_1)} \times \frac{a}{p_{hR}},
\]

Thus, the following boundary conditions are set for two domains, only one domain is needed to calculate the total load equilibrium over the entire solution domain is summed as follows by the incorporation of the cavitation term:

\[
\nabla \cdot \epsilon - \nabla \cdot \nabla \phi - \rho_{ad} = 0,
\]

where \(\phi\) is an arbitrary large positive number and \(\rho_{ad}\) is applied to Reynolds equation by Wu [20]. This method introduces the equivalent elastic modulus:

\[
E_{eq} = \frac{E_1E_2(1 + \nu_2)^2 + E_2E_1(1 + \nu_1)^2}{E_1(1 + \nu_2) + E_2(1 + \nu_1)} \times \frac{a}{p_{hR}},
\]

It is important that the above equations are applied to the appropriate domains. Thus, the following boundary conditions are defined for the total computational domain \(\Omega_c\), including the EHL domain, \(\Omega_e\), the symmetry boundary, \(\partial \Omega_e\) and the bottom boundary, \(\partial \Omega_d\), as shown in Fig. 2.

\[
\begin{align*}
\phi_{\partial \Omega_e} &= \frac{\phi_{\partial \Omega_e}}{\partial \Omega_e} + \frac{A}{\partial \Omega_e} \Omega_eY, & \text{on } \partial \Omega_c, \\
U &= V = W = 0, & \text{on } \partial \Omega_b, \\
V &= 0, & \text{on } \partial \Omega_d, \\
\phi &= \phi_0, & \text{elsewhere}.
\end{align*}
\]

The computational domain \(\Omega\) is set to 60 x 0.5 x 60 due to the symmetry and Reynolds equation is solved on the upper boundary \(\Omega_u\), where \(3 \leq X \leq 1.5\) and \(0 \leq Y \leq 0.5\). Deformation at the bottom boundary, \(\Omega_d\), is equal to zero and the boundary \(\Omega_c\) is taking care of the symmetry.

### 2.3. Load

The load equilibrium over the entire solution domain is summarized by the pressure calculation. It has no influence on the results as long as the obtained pressure is positive. When the pressure reaches to negative values at the outlet zone, the penalty parameters changes the negative quantities towards zero. This action also satisfies the condition that \(P \geq 0\) over the solution domain. The compact form of the steady state Reynolds equation (Eq. (3)) is modified as follows by the incorporation of the cavitation term:

\[
\nabla \cdot \left( \epsilon \frac{\rho_{ad}}{\partial X} \right) - \phi - P^* = 0,
\]

where \(\phi\) is an arbitrary large positive number and \(P^*\) is the load equilibrium over the entire solution domain is summed as follows by the incorporation of the cavitation term:

\[
\nabla \cdot \left( \epsilon \frac{\rho_{ad}}{\partial X} \right) - \phi - P^* = 0,
\]

3. Numerical analysis

The Reynolds Eq. (3), film thickness Eq. (6), linear elasticity Eq. (9) and force balance Eq. (13) must be solved simultaneously to obtain the pressure distribution and film thickness profile. A finite element approach was used to solve these non-linear differential equations. Bring in mind the fact that their solution are likely to exhibit undesired oscillations, particularly under high load conditions, see e.g. [14]. This instability was managed by modifying Eq. (3) with the Galerkin least square element method (GLS), presented in [21] and the isotropic diffusion technique [22]. The weak formulation of the steady state Reynolds equation is

\[
\int_{\partial \Omega} \epsilon \nabla P^* \cdot \nabla W_p^* d\Omega + \int_{\partial \Omega} \left( \beta \cdot \nabla P^* - Q \right) \nabla W_p^* d\Omega + \int_{\partial \Omega} \phi \cdot P^* W_p^* d\Omega = 0,
\]

where the first two terms represent the application of Galerkin formulation to Reynolds equation, the third describe the cavitation, the fourth represent the stabilizing GLS term, and the final terms represent the application of the isotropic diffusion technique. \(W_p^*\) represents a weighting function, the subscript \(p\) denotes the unknown pressure and \(\Omega_{ce}\) is the set of elements belonging to the calculating domain \(\Omega_c\). Some of the important parameters in Eq. (15) are obtained using the following expressions:

\[
R(P) = -\nabla \cdot \epsilon P + \beta \cdot P - Q, \quad \frac{h_{ce}(P_e)}{2\mu_{ce}} - \frac{\eta h_{ce}}{2\mu_{ce}} \frac{\xi(P_e)}{2\mu_{ce}} = \gamma \frac{\delta H}{\partial P},
\]

where \(\rho_{ad}\) represents the relative amount of isotropic diffusion with respect to the original method. \(h_{ce}\) and \(P_{ce}\) are respectively the characteristic length and the local Peclet number of the element \(e\).

The discrete form of the linear elasticity is

\[
\int_{\partial \Omega} C_{ij} e_{ij} W_{ij} d\Omega + \int_{\partial \Omega} P^* W_{ij} d\Omega = 0,
\]

where \(\tilde{D}\) represents the displacement vector, \(e_{ij}\) represents the strain tensor and \(W_{ij}\) represents the weighting functions and subscript, \(D\), denotes the displacement unknown.

Finally, the discrete form of the force balance equation reads as follow:

\[
\int_{\partial \Omega} P W_{ij} d\Omega - \frac{\eta}{2} W_{ij} = 0,
\]

where \(W_{ij}\) represents the weighting functions.
3.1. Fully coupled simulations

The model described in the preceding section was implemented in a multi-physics modeling package and used to simulate the contact between a cam and a roller follower. The parameters used to define the rheology of the lubricant are listed in Table 1. Both steady state and time-dependent simulations were conducted. Three profiled rollers with different geometries were considered in the steady state simulations, with a constant entrainment velocity, load, and radius of curvature. The Hertzian pressure solution was taken as an initial guess; see Eq. (19).

The initial value for \( H_0 \) needs to be chosen with care in order to minimize the time required to obtain the final solution. Then Reynolds Eq. (3) is coupled with the linear elasticity Eq. (9) and the Newton-Raphson method is employed to solve the determining system of equations. The force balance Eq. (13) is used to determine the updating of the \( H_0 \) value until the pressure solution, of Reynolds Eq. (3) converges.

The time dependent simulations were focused on a cam mechanism with a crowned roller. To begin with, the results obtained for the steady state case were taken as the first iteration of the time-dependent solution process. In this simulation, the radii of curvature, load, and velocity of the contacting surfaces were varied during the time. Time steps are automatically adjusted to satisfy specified tolerances of 0.001 and \( 10^{-5} \) on the relative and absolute errors, respectively. The transient EHL simulation contains 2549 time steps and lasts about 36 h and 27 min on a computer equipped with an Intel\( \text{\textregistered} \) Xeon\( \text{\textregistered} \) E5- 2650 processor. To validate the steady-state approach presented herein, it was used to simulate the same systems as considered in two previously published studies on the elliptical EHL problem, i.e., see [15,16]. In the former one of these two studies, the film thickness was obtained by means of multi-level techniques and in the latter one an inverse approach was employed for this purpose. A range of operating conditions and aspect ratios of the contact ellipse were considered, see Section 4.1.

4. Results

This section contains the summary of the results obtained from numerical simulation using the fully coupled EHL model. In Case 1 and 2 the validity of the numerical simulation is discussed. Three different types of rollers are studied in Case 3, 4 and 5, see Section 4.2. After that, effects of tilting are discussed in Section 4.3. Finally, results for simulation of cam mechanisms over an engine cycle are presented in Section 4.4.

### Table 1
Transport properties of lubricant.

<table>
<thead>
<tr>
<th>WLF parameters</th>
<th>A1 (°C)</th>
<th>C1</th>
<th>A2 (°C)</th>
<th>C2</th>
<th>B1 (GPa⁻¹)</th>
<th>B2 (GPa⁻¹)</th>
<th>qmin (GPa⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>69.81</td>
<td>11.84</td>
<td>1.68</td>
<td>60.59</td>
<td>0.213</td>
<td>11.8</td>
<td>18.05</td>
</tr>
</tbody>
</table>

### Table 2
Minimum and central film thicknesses (in micrometers) obtained using our model and those reported by Kweh [15] and Nijenbanning [16].

<table>
<thead>
<tr>
<th>Reference</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h_{\text{min}} )</td>
<td>( h_{\text{max}} )</td>
</tr>
<tr>
<td>Nijenbanning  [16]</td>
<td>1.257</td>
<td>1.430</td>
</tr>
<tr>
<td>Kweh [15]</td>
<td>1.256</td>
<td>1.341</td>
</tr>
<tr>
<td>Present model</td>
<td>1.294</td>
<td>1.495</td>
</tr>
</tbody>
</table>

4.1. Model validation

As mentioned in the preceding Section 3.1, two test cases, Case 1 and 2, that were described in publications by [15,16], respectively, were selected to be used for validation of the present steady state model. The two test cases are defined by Moes dimensionless load parameters \((M_1 = 221.56 \text{ and } M_2 = 1000)\). The corresponding Moes dimensionless lubricant parameters are \( l_1 = 12.74 \text{ and } l_2 = 5\), respectively. The entrainment velocity was set to \( u_e = 24.87 \text{ m/s} \) and the ratio of reduced radius of curvature was \( D = R_e / R_y = 0.125\). The viscosity and density of the lubricant are estimated by the same equations that was mentioned in the references, namely the Barus (exponential expression) [23] and the Dowson-Higginson [17] equations. It takes 15 iteration by the present model to reach the converged solution (relative residual error \( 10^{-6} \) and number of degree of freedom 76,000). The central and minimum film thicknesses reported in the two reference studies were predicted by means of the present model and the results are presented in Table 2. The film thicknesses predicted in our model, yields results that deviate from those reported by less than 5%. This shows that the simultaneous solution of the linear elasticity equation and Reynolds equation as used in our model yields results that agree well with conventional approaches.

4.2. Steady state simulations

As mentioned above, cylindrical bodies are profiled at their edges to reduce stress concentrations and smoothen the pressure distribution. This section describes the steady-state modeling of three different rollers with industrial applications, using the lubricant properties presented in Table 1. The third case (Case 3) considered is that of a crowned roller with \( R_e = 560 \text{ mm} \text{ and } R_y = 12.5 \text{ mm} \). The applied load is 8000 N, which correspond to a \( P_h \) value of 1.20 GPa. The entrainment velocity was specified as \( u_e = 4 \text{ m/s} \) and 0.5 m/s. Due to the symmetry, only half of the computational domain is modeled: \(-3 \leq x \leq 1.5 \text{ and } 0 \leq y \leq 0.5\). The chosen boundary conditions were \( P = 0 \) on \( \Omega_{\text{in}} \) and \( \Gamma_{\text{p}} = 0 \) on the symmetry boundary condition \( (y=0) \) and \( P = \nabla P \cdot n = 0 \) on the cavitation boundary. Fig. 3 shows contour of dimensionless pressure distribution and film thickness. The pressure distribution and film thickness along the central line in the \( Y\)-direction and \( X\)-direction are shown in Fig. 4. At the lower entrainment velocity, \( (u_e = 0.5 \text{ m/s}) \), the film thickness decreases and the predicted minimum film thickness is located at the side lobe of the contact. Also, under this condition the pressure spike decreases while the maximum pressure is higher at \( u_e = 4 \text{ m/s} \). In this condition \( (u_e = 0.5 \text{ m/s}) \) the ratio of lubricant film thickness at the inlet to the central film thickness is still higher than 11.298 which shows that the contact is not starved. For more details about the starvation effects in lubricated contact see Brikhoff and Hays [24]. Moreover, the pressure distribution for \( u_e = 0.5 \text{ m/s} \) looks much more like the Hertzian than at \( u_e = 4 \text{ m/s} \).

The forth case (Case 4) considered is a roller with a logarithmic profile. This is readily apparent in Fig. 5, which shows the contour of the dimensionless pressure distribution and film thickness. For
this case three different loads are selected, namely 1000 N, 8000 N and 11,000 N. The values for viscosity parameters, $\alpha_{\text{film}}$ and $\eta$, are chosen from Table 1 and the entrainment velocity is kept constant, $u_e=4$ m/s. The film thickness and pressure along the central line in the Y-direction and X-direction are shown in Fig. 6. It shows the variation of the film thickness according to the load variation. It is observed that the load does not much influence the film thickness. When the load rises, i.e. 11,000 N, it mostly influences the elastic deformation of the mating surfaces than the reduction of hydrodynamic film. In contrary, at low load condition, i.e. 1000 N, the
Fig. 5. Contour of dimensionless pressure distribution (left) and film thickness (right) for the logarithmic roller (Case 4).

Fig. 6. Effect of load on the pressure and film thickness, \( F = 1000 \text{N (---)} \), \( F = 8000 \text{N (—)} \) and \( F = 11000 \text{N (---)} \): Dimensionless pressure distribution and film thickness along the central line in the Y-direction (left) and X-direction (right), (Case 4).
film thickness and the pressure film seems to be close to the hydrodynamic lubrication concept.

The fifth case (Case 5) considered is a roller with a finite length roller with rounded edges where the radius starts at \( Y = 0.35 \). In this case the rounded radius, \( r \) is varied form \( r/L = 2 \) to \( r/L = 10 \). The entrainment velocity is kept constant \( u_e = 2 \text{ m/s} \). The load is selected as \( F = 3800 \text{ N} \), which correspond to \( p_0 = 1.18 \text{ GPa} \). The dimensionless pressure distribution and film thickness a long the center line in \( Y \)-direction are shown in Fig. 7. The maximum pressure and the minimum film thickness belongs to the roller with the smallest rounded radius. During the numerical simulation the film thickness has tendency to become zero or negative when \( r/L \geq 30 \) and it is difficult to obtain converged results.

### 4.3. Effects of tilting

Steady-sate simulations for three type of rollers were performed to investigate the effects of tilting angle on lubricant performance. To define the tilting, the following equation is added to the film thickness Eq. (6).

\[
H_{\text{tilt}} = Y L R_x (a_0^2 / 4 \tan(\theta))
\]  

(20)

The computing domain is changed in the axial direction to \(-L/2 \leq y \leq L/2 \) since the axial symmetry is missed. The inlet and the outlet in the \( X \)-direction is kept the same as before. The properties of the lubricant is selected from Table 1. The first case is the crowned roller. Two different loads, \( F = 2000 \text{ N} \) and \( F = 8000 \text{ N} \), are chosen. The entrainment velocity is set to \( u_e = 2 \text{ m/s} \). The tilting angle is set to \( \theta = 0.01^\circ \) and \( \theta = 0.04^\circ \). Fig. 8 shows the variation of the film thickness and pressure distribution in the axial direction for both cases. As expected for this type of roller these tilting angles does not affect the performance very much.

The second case is the roller with logarithmic profile. For this case effects of entrainment velocity and load on the lubricant performance are investigated. To begin with, the load is set to \( F = 8000 \text{ N} \) and entrainment velocities are chosen \( u_e = 0.5 \text{ m/s} \) and \( u_e = 4 \text{ m/s} \). Fig. 9 shows the variation of film thickness and pressure in the axial direction when the tilting angles are \( \theta = 0.0^\circ \), \( \theta = 0.01^\circ \) and \( \theta = 0.04^\circ \). When the tilting angle is zero the minimum film thickness and maximum pressure occurs at the both edge of the contact. However, when the tilting angle increases the minimum film occurs at the left side of the roller, see Fig. 9(c) and (e). It is readily apparent that the maximum pressure occurs at the left side of the roller as well. The higher tilting angle, the higher pressure peak, see Fig. 9(d) and (f). The entrainment velocity has strong influence on the variation of the film thickness. When the entrainment velocity decreases, the film thickness reduces, see Fig. 9(a). At the tilting situation, the film thickness reduces even more, and the position of the minimum film thickness is shifted to the left side Fig. 9(e).

The same condition as above is applied to the roller with a logarithmic profile. However, this time the load is reduced to \( F = 2000 \text{ N} \). At this condition the elastic deformation of the bodies are less than highly load case. That increases the difference of the film thickness at both end, see 10(a) and Fig. 10(c). For example, the difference between the film thickness at the two end, for \( F = 2000 \text{ N} \) and \( u_e = 4 \text{ m/s} \), is nearly 35% higher than previous condition \( (F = 8000 \text{ N} \text{ and } u_e = 4 \text{ m/s}) \), see Fig. 9(e) and Fig. 10(e). This make the slope of pressure steeper in the light load condition, see Fig. 10(b) and (d). In this situation when the entrainment velocity decrease to \( u_e = 0.5 \text{ m/s} \), the pressure peak at the left side increases and it has a steeper slope as well, see Fig. 10(f).

The third case considered is the roller with a rounded corner at the edges. In this case the entrainment velocity is kept constant \( u_e = 2 \text{ m/s} \). The loads are selected as \( F = 3800 \text{ N}, 1800 \text{ N} \) and \( 800 \text{ N} \). The rounded radius is selected as \( r = 12.7 \text{ mm} \). The length of the

![Fig. 7. Dimensionless film thickness (left) and pressure distribution (right) in the axial direction for the roller with rounded edges (Case 5).](image-url)
The tilting angle is set $\theta = 0.04^\circ$. In comparison to the roller with logarithmic shape and the roller with crowning radius, the pressure peak is sharper at the left side, see Fig. 11. As the load increases the peak becomes sharper at the both side. Like the previous case, the difference between the film thickness at both ends are higher at low load case ($F = 800$ N) and the minimum film thickness belongs to the high load condition. The location of minimum film thickness moves towards to the edge of roller (side lobe of contact) when the load increases. Also, the position of the minimum film is located at the left edge of the roller due to tilting of the roller.

4.4. Time dependent simulations

Time-dependent simulations were performed to study the variation in the pressure distribution of the cam and roller over a complete engine cycle, during which the radius of curvature, entrainment velocity, and applied load all varied. The dimensionless input parameters used in these simulations are presented in Fig. 12. The roller type is the one with the crowned radius. The lubricant properties are chosen from Table 1. During the early stage of the cycle, stage 1, the load and entrainment velocity increase while the radius of curvature is constant. The components then enter into a highly loaded state that persists for some time (stage 2). During this phase, the entrainment velocity increases linearly while the radius remain constant. At the stage 3, the load and velocity reduces with small slope. Towards the end of the engine cycle, stage 4, the load and radius reach their minima while the velocity increases modestly. At the very end of the cycle, stage 5, the velocity and radius rise to their peak values while the system is operating in a low-load state.

The variation in the central film thickness and pressure distribution over the course of a complete engine cycle in the axial and lateral direction is shown in Fig. 13. Fig. 13(a) and (b) show $P$ and $H$ at $T = 1.7$ where the load and entrainment velocity are increasing with constant radius of curvature. At $T = 2.63$ the effects of sudden reduction in the velocity can be seen in Fig. 13(c) and (d). The film thickness reduces and the pressure spike increases rapidly. Due to this rapid change, the lubricant trapped at the contact and some dynamic effects can be seen on the central film thickness in the lateral direction, see Fig. 13(d). Also, the location of the minimum film thickness moves towards the edge of the roller (side lobe of contact) due to maximum load, see Fig. 13(c). At $T = 11.63$ the behavior is mostly governed by low entrainment velocity and high load. In that situation, the pressure reaches its maximum value and the pressure spike reduces significantly, see Fig. 13(e) and (f). Fig. 13(g) and (h) show the influence of increase in the velocity, radius of curvature and load. These changes can be seen radially at $T = 15.53$ where the pressure spikes suddenly increases. At $T = 18.5$ the cams operate under low load condition where the behavior of pressure and film thickness seems to be close to the hydrodynamic lubrication concept, see Fig. 13(i) and (j). Dimensionless central film thickness and minimum film thickness over the engine cycle are depicted in Fig. 14. During the
early stage, stage 1, the film thickness decreases due to increase of load where the load reaches its maximum value. Then the film thickness reduces again due to the reduction of the velocity. At stage 2, the film thickness slightly increases, here the load and radius are constant while velocity slightly increases. At the next stage, stage 3, the film thickness becomes almost constant because the radius, velocity and load do not change. Between \(T=10.9\) to \(T=12\) the film reaches its minimum values due to drop in velocity.
and radius. However, the film thickness increases at the end due to the reduction of load. Then, at stage 4, the film slightly increases and at the final stage the film reaches to its maximum value. These variations in the film thickness proves that the velocity is the most influential parameters on the film thickness and the transient effects are relatively small. Geometry of the roller influences the lubrication performance as well. To show this, the same simulation is run with the assumption that the roller does not have any

![Graphs showing film thickness and pressure variations for different angles and velocities.](image-url)
profiling. The model is categorized as a 3D-line contact problem and two periodic boundary conditions are used instead of dirichlet boundary at the edges of the contact. The film thickness Eq. (6) is change to the following equation:

$$H(X, Y, T) = H_0(T) + X^2/2R_i + W(X, Y, T).$$

(21)

The rest of operating conditions are kept the same as before. The minimum film thickness and central film thickness over the

![Fig. 11. Roller with rounded corner ($u_e = 2\text{ m/s}$): film thickness variation for $\theta = 0.04^\circ$ (left) and pressure variation for $\theta = 0.04^\circ$ (right).](image1)

![Fig. 12. Input parameters, external load (---), velocity (---), and radius of curvature (---) for the fully coupled analysis.](image2)
Fig. 13. Dimensionless pressure and film thickness in the axial and lateral direction for different time steps, (a) Axial direction, $T = 1.7$ (b) Rolling direction, $T = 1.7$ (c) Axial direction, $T = 2.63$ (d) Rolling direction, $T = 2.63$ (e) Axial direction, $T = 11.63$ (f) Rolling direction, $T = 11.63$ (g) Axial direction, $T = 15.31$ (h) Rolling direction, $T = 15.31$ (i) Axial direction, $T = 18.5$ (j) Rolling direction, $T = 18.5$. 
engine cycle are shown in Fig. 14. These values are compared with the previous case. It is observed that the predicted film from the crowned roller has lower value than the one with line contact assumption. The main reason for this reduction is coming from the axial radius of the roller which is neglected in the 3D-line contact model. Note that the minimum film thickness in the 3D-line contact model occurs at the exit boundary while the minimum film for the crowned roller occurs at the side lobes.

5. Conclusions

A fully coupled isothermal numerical model for simulating EHL finite line contacts was developed. Finite element discretization was used in order to deal with Reynolds equation and the elastic deformation equation. Undesired oscillations and computational instability were mitigated using the Galerkin least squares element method.

In addition, this model was proven feasible for time-dependent analysis of finite EHL line contacts found in real applications such as the one typically found in a cam mechanism. The model was shown to be suitable for investigating the effects of varying contact geometry, load, and relative speed of the contacting surfaces on the lubricant film thickness and pressure distribution.

The contact between a cam and three different finite length roller followers exhibiting different profiles in the axial direction were analyzed by means of the proposed model. In general, the edge effects associated with the profile, smoothed the pressure distributions in the film. It was demonstrated that the present model can be used effectively to investigate the effects of roller geometry on pressure peaks and film thickness.

Tilting of roller can cause high pressure peaks at one side of the contact and reduce the film thickness. When the tilting angle increases the position of minimum film thickness is moved towards the end of the contact.

Neglecting the profiled shape of the roller and using the simplified assumption (line contact without perfectly shaped corner) caused an overestimate of the central and minimum film thickness in the simulation.

The new models were validated by comparing their output to representative numerical results from the literature; good agreement with the reference data was achieved in all cases.

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