Full field measurements and identification in Solid Mechanics

Depth-resolved phase imaging

J. M. Huntleya* and P. D. Ruiza

aLoughborough University, Wolfson School of Mechanical and Manufacturing Engineering, Loughborough LE11 3TU, United Kingdom

Abstract

Traditional full-field interferometric techniques (speckle, moiré, holography etc) encode the surface deformation state of the object under test in the form of 2-D phase images. Over the past 10 years, a family of related techniques (Wavelength Scanning Interferometry, Phase Contrast Spectral Optical Coherence Tomography (OCT), Tilt Scanning Interferometry and Hyperspectral Interferometry) has emerged that allows one to measure the volume deformation state within weakly-scattering objects. The techniques can be thought of as combining the phase-sensing capabilities of Phase Shifting Interferometry and the depth-sensing capabilities of OCT. This paper provides an overview of the techniques, and describes a theoretical framework based on the Ewald sphere construction that allows key parameters such as depth resolution and displacement sensitivity to be calculated straightforwardly for any given optical geometry and wavelength scan range. Finally, the related issue of robust phase unwrapping of noisy 3-D wrapped phase volumes is also described.

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1. Introduction

The use of phase information to provide data on displacement, strain or velocity fields has become widespread in both experimental solid mechanics and experimental fluid mechanics. In solid mechanics, optical techniques such as speckle, moiré, holography etc provide 2-D phase images related to the deformation state of the surface of a 3-D object. In fluid mechanics, phase contrast Magnetic Resonance Imaging (MRI) can be used to measure internal velocity fields. In this paper we review some of the recent developments in phase imaging techniques for solid mechanics applications that extend beyond the

* Corresponding author. Tel.: +44 1509 227560.
E-mail address: j.m.huntley@lboro.ac.uk

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Increasing the dimensionality of the measurements through a further spatial (or time) axis provides information that can increase the reliability of both the process of phase unwrapping, i.e. the determination of the unknown integral multiple of $2\pi$ in the measured phase, and of subsequent post-processing steps such as estimation of the elastic modulus fields. A more detailed description of the techniques and analysis presented in this paper may be found in [1].

An example of some of the issues involved in lower-dimensional phase unwrapping (i.e., 2-D spatial [2] and 1-D temporal [3] unwrapping algorithms) are shown in Fig. 1, in which data from a dynamic speckle interferometer has been deliberately undersampled along both the temporal and spatial axes to illustrate how both these approaches start to fail as the relevant Nyquist limit is approached.

The spatial unwrapping algorithm, which is based on identification of branch cuts in the 2-D phase images, has been extended into three dimensions [4-7]. The singular points that give rise to the propagation of phase unwrapping errors form singular lines in 3-D space. Provided any undersampling of the phase volumes is sufficiently localized, the lines form closed loops. An unwrapping path that passes through one of these loops will result in the propagation of a $2\pi$ phase error. By placing branch cut surfaces across the loops, a robust 3-D algorithm can be developed.

The phase singularity loops for the unwrapped phase volume shown in Fig. 1 is illustrated in Fig. 2, and the resulting unwrapped frames shown in the bottom row of Fig. 1. The increased dimensionality of the data has resulted in a clear improvement in the quality of the recovered phase distribution.

Fig. 1. Four successive phase maps, measured using speckle interferometry, from a carbon fibre sample containing a sub-surface delamination crack. Wrapped phase (top row); result of unwrapping using a branch-cut based 2-D spatial unwrapping algorithm (row 2), temporal unwrapping algorithm (row 3) and full 3-D unwrapping algorithm (bottom row).

The physical quantity encoded by the phase maps is in this case the out-of-plane displacement component. A $2\pi$ phase change represents a displacement of $\lambda/2 = 266$ nm, where $\lambda$ is the wavelength. Although the geometry of sub-surface damage can be inferred from such data, significant *a priori* knowledge must be included such as the assumed presence of a single delamination crack [8]. On the
other hand, if displacement field data is obtainable through the full sample volume, techniques such as the Virtual Fields Method can be extended into three dimensions to provide robust estimates of 3-D spatial distributions of material modulus without the need for such assumptions [9].

Fig. 2. Phase singularity loops for the full time-varying phase distribution from which the subset shown in Fig. 1 was taken. The vertical axis is frame number.

2. Interferometric techniques for depth-resolved phase imaging

An alternative to the ‘two spatial plus one time’ axes involves the acquisition of data that allows phase volumes to be reconstructed along all three spatial coordinates. Several methods have been developed which can be regarded as a combination of the depth-sensing capabilities of Optical Coherence Tomography and the phase-sensitive capabilities of traditional 2-D full-field interferometric techniques. This section provides a brief overview of the main families that have been developed to date to measure internal displacement fields within weakly-scattering transparent materials.

2.1. Low Coherence Speckle Interferometry (LCSI) [10,11]

This approach, which can also be regarded as a phase-sensitive version of time-domain OCT, involves the use of a low coherence light source such as a superluminescent LED. As a result, an interference signal is produced only from those parts of the sample where the object and reference wave path lengths are matched to within the coherence length of the light source. Conventional 2-D phase difference maps are produced that thus encode the displacement field for one ‘slice’ within the sample. LCSI has some attractive features, in particular the experimental simplicity, however it only measures in two dimensions so that an additional scanning device is required to obtain a full 3-D phase volume. Furthermore, the parts of the sample that fall outside the coherence ‘gate’ contribute a dc offset to the recorded intensity distribution, which reduces significantly the number of gray levels available for the interference signal.

2.2. Wavelength-Scanning Interferometry (WSI) [12,13]

In WSI, light scattered by different layers within the material is combined with a reference wave. The optical path difference between object and reference wave varies linearly with depth of the scattering
layer within the sample. Therefore the frequency of the interference signal produced by tuning the wavenumber of the laser also varies linearly with depth. The signals from different scattering layers can therefore be separated from one another by spectral analysis of the time-varying interference signal measured at each pixel in the image plane. 3-D displacement fields can be determined by measuring phase changes in the Fourier domain between two successive scans of the sample. WSI has the benefit of higher signal-to-noise ratio than LCSI, and is intrinsically a 3-D technique. However, a tunable laser source is required and the sample needs to be kept mechanically stable for the duration of the scan.

2.3. Tilt-Scanning Interferometry (TSI) [14]

In Tilt Scanning Interferometry, depth-resolved displacements are measured by tilting the illumination angle during the acquisition of image sequences. The depth-encoding frequency shift can be regarded as coming from the Doppler shift of the photons reflected from the tilting mirror. As in WSI, spectral analysis of the intensity signal at a given camera pixel provides depth-resolved information, the magnitude relating to the specimen structure and the phase change relating to the relevant displacement component. Fig. 3 shows the setup for bending beam experiments, and sample wrapped phase maps from proof of principle experiments described in [14]. The top row of Fig. 3 shows the in-plane (x axis) phase-change distribution for slices within the beam starting at the object surface $z = 0$ mm (left) in steps of -1.74 mm down to $z = -5.22$ mm (right). Good agreement was achieved between the experimentally-determined displacement fields and a finite element model. TSI has similar stability requirements to WSI. It does however have the attractive feature of working at a single wavelength, thereby avoiding the need for an expensive tunable laser and side-stepping errors due to dispersion.

![Fig. 3. (Left) Schematic view of an epoxy resin beam undergoing 3-point bending; (right) in-plane (top row) and out-of-plane (bottom row) wrapped phase-change distribution for different slices within the beam, measured using TSI. Black represents -$\pi$ and white +$\pi$. Fringe spacing is equivalent to ~0.38\(\mu\)m and ~0.15\(\mu\)m for in-plane and out-of-plane sensitivity, respectively.](image)

2.4. Phase Contrast Spectral OCT [15,16]

Spectral OCT involves illumination with a sheet of light extending into the sample from the surface. The intersection of the sheet with the sample forms a line, which is imaged onto a 2-D sensor through a diffraction grating. The resulting images have one spatial axis and one wavenumber axis, the latter being converted to a second spatial coordinate by Fourier transformation. As with low-coherence interferometry, therefore, some form of scanning is required to measure 3-D volumes. In [15] a method was presented to measure out-of-plane displacement fields from the phase information in the spectral OCT images, and this was extended in [16] to the measurement of both in-plane and out-of-plane
displacement fields. An example of the application of the technique to deformation measurement with a pig’s cornea after a small change in the intraocular pressure is shown in Fig. 4.

Fig. 4. Phase change measured on a slice through a pig’s cornea after a change in the intraocular pressure [15].

2.5. Hyperspectral Interferometry (HSI) [17]

Hyperspectral interferometry is a recently-proposed method for the measurement of surface profiles. By using a broad-band light source and hyperspectral imaging system, a set of interferograms at different wavenumbers are recorded simultaneously on a high resolution image sensor. These are then assembled to form a three-dimensional intensity distribution. By Fourier transformation along the wavenumber axis, an absolute optical path difference is obtained for each pixel independently of the other pixels in the field of view. In effect HSI can be considered to be a full-field version of spectral OCT. The main limitation is the limited spatial resolution, however the fact that all the data is acquired in a single shot means that fringe movement due to vibration can be effectively frozen given a sufficiently short exposure duration.

3. Depth-resolved techniques viewed as linear filtering operations

A 3-D spatial frequency domain representation of the techniques was introduced in [18]. Although some simplifying assumptions are required, the benefit of the model is that it allows all the techniques to be described within a common unifying framework, and the key features such as depth resolution and displacement sensitivity to be derived from a simple geometric construction known as the Ewald sphere.

Consider a weakly scattering, non-dispersive material of complex refractive index \( n'(r) = n + \Delta n(r) \), with small variations \( \Delta n(r) \) around a uniform index \( n \) and \( r \) a 3-D position vector as shown in Fig. 5(a). A small volume \( V \) in this material is illuminated by a plane wave \( \exp(ik_o \cdot r) \) with wave vector \( k_o \) and amplitude \( A_o \). The scattered field is measured at a great distance \( R \) from \( V \), along the observation wave vector \( k_o \), where \( k_o = 2\pi/\lambda \) and \( \lambda \) represents the wavelength in the material.

Using scalar diffraction theory, and neglecting multiple scattering effects, it has been shown that the measured far-field amplitude \( U \) is given by a volume integral of the scattering potential \( \Phi(r) \), weighted by a phase factor \( \exp(-iK \cdot r) \) which arises from the variation of optical path length within the sample:

\[
U(K) \propto \int \Phi(r) \exp(-iK \cdot r) d^3r.
\]

\( K = k_o - k_i \) is the scattering vector defined in terms of the observation and illumination wave vectors, and the scattering potential \( \Phi(r) \) represents the object microstructure within \( V \) and is related in turn to \( \Delta n(r) \).
The significance of Eqn. (1) is that the measured amplitude is proportional to one component of the 3-D Fourier transform of the scattering potential. Physically, the microstructure can be regarded as a superposition of sets of 3-D parallel sinusoidal fringes known in holography and crystallography as Bragg planes. \( U \) measures the amplitude of the particular set of Bragg planes that have their normal along \( K \) (i.e. along the bisector of the illumination and observation directions) and have a pitch of \( 2\pi/|K| \). In what follows, we use the term ‘\( K \)-space’ to indicate the 3-D Fourier transform of the scattering potential for a given sample microstructure. A scattering vector or set of scattering vectors can be drawn on a \( K \)-space diagram to indicate which Fourier components are measurable by the interferometer.

3.1. Methods viewed as linear filtering operations

In principle, if one were to measure \( U(K) \) on a sufficiently fine and extended 3-D mesh of \( K \) values, Eqn. (1) shows that the required spatial variation in scattering potential could be computed as

\[
\Phi(r) \propto \mathcal{F}^{-1}[U(K)],
\]

where \( \mathcal{F}^{-1} \) is the 3-D inverse Fourier transform operator. In practice, however, one can only access a limited subset of \( U(K) \). If the illumination consists of a single plane wave of wave vector \( k_i \), all Fourier components of the scattering potential accessible by scattering at different observation directions \( k_o \) are limited to those located on the surface of a sphere described by the arrowhead of the scattering vector \( K \). This is known as the Ewald sphere for the specific wavelength \( \lambda \); it has radius \( k \) and is centred at \( k = -k_i \) (see Fig. 5(b)).

The region of \( K \)-space that is accessible by a given measurement technique can be specified by a window function \( W(K) \) that is non-zero only wherever a valid measurement can be made. The reconstructed scattering potential is then given by

\[
\Phi'(r) \propto \mathcal{F}^{-1}[U(K) \cdot W(K)].
\]
The relationship between $\Phi'(\mathbf{r})$ and $\Phi(\mathbf{r})$ follows from the convolution theorem as

$$\Phi'(\mathbf{r}) \propto \Phi(\mathbf{r}) \otimes H(\mathbf{r}),$$  \hspace{1cm} (4)

where $\otimes$ denotes convolution, and where $H(\mathbf{r})$ is the 3-D impulse response or point spread function of the measurement apparatus which is related to $W(\mathbf{K})$ as follows:

$$H(\mathbf{r}) \propto 3^{-1}[W(\mathbf{K})].$$  \hspace{1cm} (5)

Eqns. (3)-(5) express the reconstruction problem as a linear, shift invariant, filtering operation. $H(\mathbf{r})$ is a key function for all the depth-resolving techniques since it determines both their spatial resolution and their sensitivity. These two aspects are now considered individually in Sections 3.2 and 3.3, before moving onto one specific example (WSI) in Section 3.4.

3.2. Relationship between $W(\mathbf{K})$ and spatial resolution

The relationship between $W(\mathbf{K})$ and $H(\mathbf{r})$ given by Eqn. (5) is illustrated in Fig. 6. In this example, $W(\mathbf{K})$ is non-zero only near to the $\mathbf{K}$-space origin, which physically corresponds to measurements made close to the forward scattering direction. If we denote the characteristic dimensions of $W$ along the $K_x$, $K_y$, and $K_z$ directions by $\Delta K_x$, $\Delta K_y$, and $\Delta K_z$, respectively, the spatial extent of the point spread function along the $x$-, $y$- and $z$-axes, i.e. the resolution of the tomographic imaging system, is given to a first approximation by:

$$\delta x = \gamma \frac{2\pi}{\Delta K_x}, \delta y = \gamma \frac{2\pi}{\Delta K_y}, \delta z = \gamma \frac{2\pi}{\Delta K_z},$$  \hspace{1cm} (6)

where $\gamma$ is a constant that reflects the influence of the shape of the window function. For example, if $W(\mathbf{K})$ is a rectangular cuboid, then $\gamma = 2$ defines the full width of the point spread function along each axis as measured between the zero-crossing points.

3.3. Relationship between $W(\mathbf{K})$ and displacement sensitivity

The point spread function $H(\mathbf{r})$ is in general a complex function of position. Fig. 7 illustrates this point with the window function $W = W_1(\mathbf{K})$ from Fig. 6, in which the real and imaginary parts of $H = H_1(\mathbf{r})$ are displayed along with the equivalent representation in terms of magnitude and phase. Of direct interest for depth-resolved displacement measurement is the phase variation within the point spread function since this dictates the measured phase change at a given point in the tomographic reconstruction as a scattering point moves. The phase gradient in (a) is however very low because all the measured frequency components are clustered around the origin of $\mathbf{K}$-space. As a result, measurements made with any interferometric depth-resolving technique close to the forward-scattering direction will have poor displacement sensitivity.

Suppose the interferometer is modified to allow it to measure a different region of $\mathbf{K}$-space, defined by a second window function, $W = W_2(\mathbf{K})$, that is identical to $W_1$ except for a translation by the vector $\mathbf{K}_c$ as shown in Fig. 7(b), i.e.

$$W_2(\mathbf{K}) = W_1(\mathbf{K} - \mathbf{K}_c).$$  \hspace{1cm} (7)

By the Fourier shift theorem, the resulting point spread function $H = H_2(\mathbf{r})$ is related to $H_1$ as follows:
\[ H_2(r) = \exp(iK_c \cdot r)H_1(r). \] (8)

Fig. 6. Relationship between the K-space window function \( W(K) \) and real-space point spread function \( H(r) \).

Fig. 7. Schematic illustration of the real and imaginary parts, and magnitude and phase, of the point spread function \( H(r) \) for a window function \( W(K) \) (a) centred on the origin of K-space and (b) shifted by \( K_c \).

The dimensions of \( H_2 \) are identical to \( H_1 \), and therefore the spatial resolutions of the original and modified interferometers are identical to one another. As a result of the \( \exp(iK_c \cdot r) \) term in Eqn. (8), however, the real and imaginary parts of \( H_2 \) are now modulated by parallel sinusoidal fringes orientated normal to \( K_c \) and with spacing \( 2\pi/K_c \), as shown in Fig. 7(b). The corresponding phase distribution within \( H_2 \) has a gradient of \( 2\pi \) per cycle of these fringes. The tomographic reconstruction of a scattering point will therefore show a phase change of \( 2\pi \) for a movement of \( 2\pi/K_c \) along the \( -K_c \) direction.
This should come as no surprise once it is realized that the scattering vector $\mathbf{K}$, and the sensitivity vector from the speckle interferometry literature are (if one neglects the sign) the same quantity. The beauty of the Ewald sphere construction for interpreting depth-resolved displacement field measurements is that it shows in a simple pictorial way:

(a) the spatial frequencies of the sample microstructure that may be imaged:
(b) the spatial resolution along all three axes (from the window function shape);
(c) the displacement component (from the direction of $\mathbf{K}_c$); and
(d) the displacement sensitivity (from the magnitude of $\mathbf{K}_c$).

3.4. Application to WSI

A Wavelength Scanning Interferometer and its Ewald spheres are shown schematically in Fig. 8(a) and (b). If the illuminating beam is collimated vertically downwards, then $\mathbf{k}_i$ is aligned along the $-z$ direction. In common with all far-field optical instruments, only a fraction of the scattered field is collected by the entrance pupil of the interferometer, defined by the object-space numerical aperture of the system $N_A = n \sin(\alpha)$, where $\alpha$ is the half-angle subtended by the cone of rays accepted by the aperture from a point in the object. The measurable $\mathbf{k}_o$ for this interferometer at the start of a wavelength scan therefore covers a range that is represented by the bold curve on the inner continuous circle of Fig. 8(b). The allowable $\mathbf{K}$ vectors are produced by adding this range of $\mathbf{k}_o$ to the constant $-\mathbf{k}_i$ resulting in the bold curve on the inner dashed circle. In three dimensions, the allowable $\mathbf{K}$ vectors lie on a cap at the top of the Ewald sphere.

As the wavelength scan proceeds, the bold curve sweeps out the region of $\mathbf{K}$-space shown in grey in Fig. 8(b), which is therefore the window function $W(\mathbf{K})$. The scattering vector $\mathbf{K}_c$ is shown in Fig. 8(c). From the orientation of $\mathbf{K}_c$ this interferometer measures the out-of-plane displacement component $u_z$. The magnitude of $\mathbf{K}_c$ ($|\mathbf{K}_c| = 4\pi n / \lambda_c$) shows that the displacement sensitivity is $\lambda_c/2n$ per $2\pi$ phase change.

Fig. 8(c) also shows the window function dimensions $\Delta K_x$, $\Delta K_y$, $\Delta K_z$. $\Delta K_z$ is $2\Delta k = 2n\Delta k$. Using the relations in Eqn. (6), the axial resolution is therefore $\delta z = \gamma \pi / n \Delta k \approx \gamma \lambda_c^2 / 2n \Delta \lambda$. $\Delta K_x$ and $\Delta K_y$ both take the value $2nk \sin \alpha$, and therefore the lateral resolution is $\delta x = \delta y = \gamma \pi / nk_c \sin \alpha = \gamma \lambda_c / 2n \sin \alpha$ which
is, within a numerical factor, the usual Rayleigh criterion for the diffraction-limited lateral resolution of an imaging system.

4. Conclusions

This paper has summarized some of the recent developments in depth-resolved phase imaging in weakly-scattering materials. Increasing the dimensionality of the measurements from the usual two dimensions has clear benefits both for the essential phase unwrapping step and for subsequent post-processing steps such as elastic modulus reconstruction algorithms. All the described techniques can be considered within a common theoretical framework based on the Ewald sphere construction, which allows straightforward prediction – and hence optimization – of the spatial resolution and displacement sensitivity of a given interferometer configuration.

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