MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects

B. Ganga\textsuperscript{a}, S. Mohamed Yusuff Ansari\textsuperscript{b}, N. Vishnu Ganesh\textsuperscript{c}, A.K. Abdul Hakeem\textsuperscript{c,*}

\textsuperscript{a} Department of Mathematics, Providence College for Women, Coonoor - 643 104, India
\textsuperscript{b} Department of Mathematics, Jamal Mohamed College, Trichy - 642 020, India
\textsuperscript{c} Department of Mathematics, Sri Ramakrishna Mission Vidyalaya College of Arts & Science, Coimbatore - 641 020, India

Received 29 December 2014; received in revised form 20 April 2015; accepted 21 April 2015

Abstract

A mathematical analysis has been carried out to investigate the effects of internal heat generation/absorption, viscous and ohmic dissipations on steady two-dimensional radiative MHD boundary-layer flow of a viscous, incompressible, electrically conducting nanofluid over a vertical plate. A system of governing nonlinear PDEs is converted into a set of nonlinear ODEs by suitable similarity transformations and then solved analytically using HAM and numerically by the fourth order Runge–Kutta integration scheme with shooting method. The effects of different controlling parameters on the dimensionless velocity, temperature and nanoparticle volume fraction profiles are discussed graphically. The reduced Nusslet number and the local Sherwood number are also discussed graphically. It is found that the presence of viscous dissipation, heat generation and magnetic field accelerates the temperature and decelerates the nanosolid volume fraction profile. Furthermore, comparisons have been made with benchmark solutions for a special case and obtained a very good agreement.

Keywords: Heat generation/absorption; Homotopy analysis method; Nanofluid; Ohmic dissipation; Viscous dissipation; Vertical plate

1. Introduction

Nanofluids are suspensions of nanoparticles in fluids which was introduced by Choi [1] that show significant enhancement of their properties at modest nanoparticle concentrations. Many of the publications on nanofluids are about understanding their behaviour so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications such as nuclear reactors, transportation, electronics as well as biomedicine and food. This concept attracted various researchers towards nanofluids, and various theoretical and experimental studies have been done to find the thermal properties of nanofluids. Boungiorno et al. [2] studied the thermal conductivity of...
of nanofluids experimentally. The same author proposed an analytical model for convective transport in nanofluids taking into the account of Brownian diffusion and thermophoresis [3].


Magnetohydrodynamic boundary layer flow is of considerable interest due to its wide usage in industrial technology and geothermal application, high temperature plasmas applicable to nuclear fusion energy conversion, MHD power generation systems and liquid metal fluids. Due to its wide range of applications, the following researchers have investigated the magnetic field effect on the fluid flow problems [13–24]. Very recently, Abdul Hakeem et al. [25] studied the magnetic field effect on second order slip flow of single phase nanofluid over a stretching/shrinking sheet.

The interaction of natural convection with thermal radiation has increased greatly during the last decade due to its importance in many practical involvements. When free convection flows occur at high temperature, radiation effects on the flow become significant. Radiation effects on the free convection flow are important in context of space technology, processes in engineering areas occur at high temperature. Based on these applications, the following researchers have investigated the magnetic field effect on the fluid flow problems [13–24]. Very recently, Abdul Hakeem et al. [25] studied the magnetic field effect on second order slip flow of single phase nanofluid over a stretching/shrinking sheet. Recently, Turkyilmazoglu and Pop [28] studied the thermal radiation effects on the flow of single phase nanofluid over a infinite vertical plate.

Viscous dissipation is quite often a negligible effect, but its contribution might become important when the fluid viscosity is very high. It changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. Anjali devi and Ganga [29] studied the effects of viscous and Joules dissipation on MHD Flow past a stretching porous surface embedded in a porous medium for ordinary fluid. The effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluid over a moving flat plate were studied by Motsumi and Makinde [30]. Very recently, Makinde and Mutuku [31] investigated the hydromagnetic thermal boundary layer of nanofluids over a convectively heated flat plate with viscous dissipation and Ohmic heating effects.

The study of heat generation or absorption effects is very important in cooling processes. Although, exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behavior for most physical situations. Ahmed et al. [32] investigated the effects of heat source/sink on the boundary layer flow of single phase nanofluid over a stretching tube. Very recently, Akilu and Narahari [33] studied the effects of internal heat generation/absorption on natural convection flow of a nanofluid over an inclined plate numerically.

The main goal of this paper is to analyze the effects of internal heat generation/absorption, viscous and ohmic dissipations on steady two-dimensional radiative MHD boundary-layer flow of nanofluid over a vertical plate both analytically and numerically. The analytical solutions are obtained using HAM and the fourth order Runge–Kutta method along with shooting technique is used to find the numerical solutions for the physical problem.

2. Formulation of the problem

We consider the steady two-dimensional boundary layer flow of a nanofluid over vertical plate in the presence of magnetic field intensity, thermal radiation, viscous dissipation, Ohmic dissipation and volumetric rate of heat generation/absorption. We select a coordinate frame in which the x-axis is aligned vertically upwards. We consider a vertical plate at y = 0. At this boundary, the temperature T and the nanoparticle volume fraction φ take constant values T_w and φ_w respectively. The temperature T and the nanoparticle volume fraction of the nanofluid φ take values T_∞ and φ_∞ respectively at y → ∞. We consider a steady state flow. We also consider influence of a constant magnetic field strength B_0 which is applied normally to the plate. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field. Under the above assumptions, the boundary layer equations

Please cite this article in press as: Ganga B, et al. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society (2015), http://dx.doi.org/10.1016/j.jnnms.2015.04.001
governing the flow, thermal and concentration field can be written in dimensional form as [7].

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

\[
\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \rho_f \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \sigma B_0^2 u + \left[ (1 - \phi_\infty) \rho_f \beta g (T - T_\infty) - \left( \rho_p - \rho_f \phi_\infty \right) g (\phi - \phi_\infty) \right]
\]  

(2)

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T + \frac{\mu}{\rho c} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c} u^2 + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left( \frac{D T}{T_\infty} \right) \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho c)_f} \left( \frac{\partial q_r}{\partial y} \right) + \frac{Q}{(\rho c)_f} (T - T_\infty)
\]  

(3)

\[
\frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \left( \frac{D T}{T_\infty} \right) \left( \frac{\partial^2 T}{\partial y^2} \right)
\]  

(4)

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) directions respectively, \( p \) is the fluid pressure, \( \rho_f \) is the density of base fluid, \( \rho_p \) is the nanoparticle density, \( \rho \) is the density of nanofluid, \( c \) is specific heat of nanofluid at constant pressure, \( \mu \) is the absolute viscosity of the base fluid, \( \alpha = \frac{k}{\rho c} \) is the thermal diffusivity of the base fluid, \( \tau = \frac{(\rho c)_p}{(\rho c)_f} \) is the ratio of nanoparticle heat capacity and the base fluid heat capacity, \( \phi \) is the local solid volume fraction of the nanofluid, \( \beta \) is volumetric thermal expansion coefficient of the base fluid, \( D_B \) is the Brownian diffusion coefficient, \( D_T \) is the thermophoretic diffusion coefficient, \( T \) is the local temperature and \( g \) is the acceleration due to gravity, \( B_0 \) is the constant magnetic field and \( Q \) is the heat generation/absorption coefficient.

The boundary conditions are taken to be,

\[
\begin{align*}
u &= 0, \quad v = 0, \quad T = T_w, \quad \phi = \phi_w \quad \text{at} \quad y = 0, \\
u &= v = 0, \quad T \to T_\infty, \quad \phi \to \phi_\infty \quad \text{as} \quad y \to \infty.
\end{align*}
\]  

(5)  

(6)

The radiative heat flux \( q_r \) is described by Roseland approximation [34,35] such that

\[
q_r = -\frac{4\sigma^*}{3\delta} \frac{\partial T^4}{\partial y}
\]  

(7)

where \( \sigma^* \) and \( \delta \) are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow are sufficiently small so that the \( T^4 \) can be expressed as a linear function after using Taylor series to expand \( T^4 \) about the free stream temperature \( T_\infty \) and neglecting higher-order terms. This result is the following approximation:

\[
T^4 \cong 4T^3_\infty \frac{T}{T_\infty} - 3T^4_\infty.
\]  

(8)

Using (7) and (8) in (3), we obtain

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T - \frac{\mu}{\rho c} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c} u^2 + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left( \frac{D T}{T_\infty} \right) \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho c)_f} \left( \frac{16\sigma^*}{3\delta} \frac{T^3_\infty}{T^2} \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q}{(\rho c)_f} (T - T_\infty).
\]  

(9)

3. Similarity transformations

The following quantities are introduced to transform Eqs. (2), (4) and (9) into ordinary differential equations [7].

\[
\eta = \frac{y}{x} Ra x^{1/4}, \quad \psi = \alpha Ra x^{1/4} s(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad f(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}
\]  

(10)
with the local Rayleigh number which is defined as

$$ Ra_x = \frac{(1 - \phi_\infty)g\beta(T_w - T_\infty)x^3}{v\alpha} $$

(11)

and the stream function $\psi(x, y)$ is defined such that

$$ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} $$

(12)

So, the continuity equation Eq. (1) is identically satisfied. After some algebraic manipulation, the momentum, energy and the solid volume fraction equations are obtained as follows,

$$ s''' + \frac{1}{4Pr}(3ss'' - 2s'^2 - 4M\sqrt{Pr}s') + \theta - Nr f = 0 $$

(13)

$$ \left(1 + \frac{4N}{3}\right)\theta'' + \frac{3}{4}s\theta' + Nb f' \theta' + Nt\theta'^2 + Ec s'^2 + \frac{EcM}{\sqrt{Pr}} s^2 + \lambda \sqrt{Pr} \theta = 0 $$

(14)

$$ f'' + \frac{3}{4}Le s f' + \frac{Nt}{Nb}\theta'' = 0 $$

(15)

where Primes denote differentiation with respect to $\eta$ and the non-dimensional parameters, Prandtl number (Pr), Buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Lewis number (Le), radiation parameter ($N$), Eckert number (Ec), heat generation or absorption parameter ($\lambda$) and Magnetic parameter ($M$), are defined as follows,

$$ Pr = \frac{v}{\alpha}, \quad Nr = \frac{(\rho_p - \rho_f)\phi)(\phi_w - \phi_\infty)}{\rho_f \beta(T_w - T_\infty)(1 - \phi_\infty)}, \quad Nb = \frac{(\rho c)_p D_B (\phi_w - \phi_\infty)}{(\rho c)_f \alpha}, $$

$$ Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \alpha T_\infty}, \quad Le = \frac{\alpha}{D_B}, \quad M = \frac{\sigma B_0^2 x^\frac{1}{2}}{\rho_f \sqrt{(1 - \phi_\infty)g\beta(T_w - T_\infty)}}, $$

$$ N = \frac{4\sigma^* T_\infty^3}{k\delta}, \quad Ec = \frac{(1 - \phi_\infty)g\beta x^2}{c} \quad \text{and} \quad \lambda = \frac{Q x^\frac{1}{2}}{(\rho c)_f \sqrt{(1 - \phi_\infty)g\beta(T_w - T_\infty)}}. $$

The corresponding boundary conditions are as follows,

$$ s(\eta) = 0, \quad s'(\eta) = 0, \quad \theta(\eta) = 1, \quad f(\eta) = 1 \quad \text{at} \ \eta = 0 $$

$$ s'(\eta) = 0, \quad \theta(\eta) = 0, \quad f(\eta) = 0 \quad \text{as} \ \eta \rightarrow \infty. $$

(16)

(17)

A quantities of practical interest are the Nusselt number $Nu$ and Sherwood number $Sh$ defined by

$$ Nu = \frac{xq_w''}{k(T_w - T_\infty)} $$

(18)

$$ Sh = \frac{xq_m''}{D_B (\phi_w - \phi_\infty)} $$

(19)

where $q_w''$ and $q_m''$ are the wall heat flux and mass flux. The reduced local Nusselt number $Nur$ in the presence of thermal radiation and reduced local Sherwood number $Shr$, can be introduced and represented as follows,

$$ Nur = Ra_x^{1/4} Nu = - \left(1 + \frac{4N}{3}\right) \theta'(0) $$

$$ Shr = Ra_x^{1/4} Sh = - f'(0). $$

Please cite this article in press as: Ganga B, et al. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society (2015), http://dx.doi.org/10.1016/j.jnms.2015.04.001
4. Analytical solution by homotopy analysis method

The Eqs. (13)–(15) are solved under the corresponding boundary conditions (16) and (17) by using HAM. For HAM solutions, we choose the initial guesses and auxiliary linear operators in the following form:

\[ s_0(\eta) = 1 - e^{-\eta} - \eta e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}, \quad f_0(\eta) = e^{-\eta} \]

\[ L_1(s) = s''' - s', \quad L_2(\theta) = \theta'' + \frac{\theta}{2}, \quad L_3(f) = f'' + f \]  \hfill (20)

and

\[ L_1(c_1 + c_2 e^\eta + c_3 e^{-\eta}) = L_2(c_1 e^\eta + c_2 e^{-\eta}) = L_3(c_1 e^\eta + c_2 e^{-\eta}) = 0. \]

\hfill (22)

And \( c_1, c_2, \) and \( c_3 \) are constants and \( p \in [0, 1] \) denotes the embedding parameter and \( h_1, h_2, \) and \( h_3 \) indicate the non-zero auxiliary parameters. We then construct the following problems:

Zeroth-order deformation problems,

\begin{align*}
(1 - p)L_1[s(\eta, p) - s_0(\eta)] &= ph_1N_1[s(\eta, p), \theta(\eta, p), f(\eta, p)] \\
(1 - p)L_2[\theta(\eta, p) - \theta_0(\eta)] &= ph_2N_2[s(\eta, p), \theta(\eta, p), f(\eta, p)] \\
(1 - p)L_3[f(\eta, p) - f_0(\eta)] &= ph_3N_3[s(\eta, p), \theta(\eta, p), f(\eta, p)]
\end{align*}

\hfill (23)

\hfill (24)

\hfill (25)

\[ s(0, p) = 0, \quad s'(0, p) = 1, \quad s'(\infty, p) = 0 \]

\hfill (26)

\[ \theta(0, p) = 1, \quad \theta(\infty, p) = 0 \]

\hfill (27)

\[ f(0, p) = 1, \quad f(\infty, p) = 0 \]

\hfill (28)

and

\[ N_1[s(\eta, p), \theta(\eta, p), f(\eta, p)] = s'''(\eta, p) + \frac{1}{4 Pr} \left( 3s(\eta, p)s''(\eta, p) - 2s^2(\eta, p) - 4M Ec s''(\eta, p) \right) + \theta(\eta, p) + Nr f(\eta, p) \]

\hfill (29)

\[ N_2[s(\eta, p), \theta(\eta, p), f(\eta, p)] = \left( 1 + \frac{4N}{3} \right) \theta''(\eta, p) + \frac{3}{4} s(\eta, p) \theta'(\eta, p) + h f'(\eta, p) \theta'(\eta, p) + Nt \theta^2(\eta, p) + Ec s''(\eta, p) + \frac{Ec M}{\sqrt{Pr}} s^2(\eta, p) + \lambda \sqrt{Pr} \theta(\eta, p) \]

\hfill (30)

\[ N_3[s(\eta, p), \theta(\eta, p), f(\eta, p)] = f'''(\eta, p) + \frac{3}{4} Les(\eta, p) f'(\eta, p) + \frac{Nt}{Nb} \theta''(\eta, p) \]

\hfill (31)

For \( p = 0 \) and \( p = 1 \), we have,

\[ s(\eta, 0) = s_0(\eta), \quad s(\eta, 1) = s(\eta) \]

\hfill (32)

\[ \theta(\eta, 0) = \theta_0(\eta), \quad \theta(\eta, 1) = \theta(\eta) \]

\hfill (33)

\[ f(\eta, 0) = f_0(\eta), \quad f(\eta, 1) = f(\eta). \]

\hfill (34)

Due to Taylor’s series with respect to \( p \), we have,

\[ s(\eta, p) = s_0(\eta) + \sum_{m=1}^{\infty} s_m(\eta) p^m \]

\hfill (35)

\[ \theta(\eta, p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \]

\hfill (36)

\[ f(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \]

\hfill (37)

\[ s_m(\eta) = \frac{1}{m!} \frac{\partial^m s(\eta, p)}{\partial p^m} \]

\hfill (38)
\[\theta_m(\eta) = \frac{1}{m!} \frac{\partial^m (\theta(\eta, p))}{\partial p^m} \]  
\[f_m(\eta) = \frac{1}{m!} \frac{\partial^m (f(\eta, p))}{\partial p^m} \]

and thus, \(m\)th-order deformation problems

\[L_1[s_m(\eta) - \chi_m s_{m-1}(\eta)] = h_1 R_m^s(\eta)\]  
\[L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_2 R_m^\theta(\eta)\]  
\[L_3[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_3 R_m^f(\eta)\]

and

\[s_m(0) = s'_m(0) = s''_m(\infty) = 0\]  
\[\theta_m(0) = \theta_m(\infty) = 0\]  
\[f_m(0) = f'_m(\infty) = 0\]

where,

\[R_m^s = s''_{m-1} + \frac{1}{4Pr} \left( \frac{3}{2} \sum_{i=0}^{m-1} s_{m-1-i} s''_i - 2 \sum_{i=0}^{m-1} s_{m-1-i} s'_i - 4M \sqrt{Pr} s'_{m-1} \right) + \theta_{m-1} - Nr f_{m-1}\]

\[R_m^\theta = \left( 1 + \frac{4N}{3} \right) \theta''_{m-1} + \frac{3}{4} \sum_{i=0}^{m-1} s_{m-1-i} \theta'_i + Nb \sum_{i=0}^{m-1} f'_{m-1-i} \theta'_i + Nt \sum_{i=0}^{m-1} \theta''_{m-1-i} \theta'_i + Ec \sum_{i=0}^{m-1} s''_{m-1-i} s'_i + \sqrt{Pr} \sum_{i=0}^{m-1} s'_{m-1-i} s'_i + \lambda \sqrt{Pr} \theta_{m-1}\]

\[R_m^f = f''_{m-1} + \frac{3}{4} Le \sum_{i=0}^{m-1} s_{m-1-i} f'_i + \frac{Nt}{Nb} \theta''_{m-1}\]

and

\[\chi_m = \begin{cases} 0 & (m \leq 1) \\ 1 & (m > 1) \end{cases} \]

which \(h\) is chosen in such a way that these three series are convergent at \(p = 1\), therefore we have,

\[s(\eta) = s_0(\eta) + \sum_{m=1}^{\infty} s_m(\eta)\]

\[\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)\]

\[f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)\]

4.1. Convergence of HAM

As pointed by Liao [36], the convergence rate of approximation for the HAM solution strongly depends on the value of auxiliary parameter ‘\(h\)’. Fig. 1 clearly depict that the range for admissible value of \(h\) is \(-1.1 \leq h \leq -0.1\). In our calculation for this case, it clearly indicates that \(h = -0.3\) for whole region \(\eta\).
5. Numerical method for solution

The nonlinear coupled differential equations (13)–(15) along with the boundary conditions (16) and (17) form a two point boundary value problem and is solved using shooting technique together with fourth order Runge–Kutta integration scheme by converting it into an initial value problem. In this method we have to choose a suitable finite value of $\eta \to \infty$, say $\eta_\infty$. We set following first order system:

$$
y_1' = y_2,
$$
$$y_2 = y_3,
$$
$$y_3' = -\frac{1}{4Pr}(3y_1y_3 - 2y_2^2 - 4M\sqrt{Pr}y_2) - y_4 + N_ry_6,
$$
$$y_4' = y_5,
$$
$$y_5' = \left(\frac{-3}{4}y_1y_5 - N_by_7y_5 - N_ty_5^2 - \lambda\sqrt{Pr}y_4 - Ec\frac{M}{\sqrt{Pr}}y_2^2\right)\left(\frac{3}{3 + 4N}\right),
$$
$$y_6' = y_7,
$$
$$y_7' = -\frac{3}{4}L_ey_1y_7 - N_t\frac{y_5}{N_b},
$$
with the boundary conditions

$$y_1(0) = 0, \quad y_2(0) = 0, \quad y_4(0) = 1 \quad \text{and} \quad y_6(0) = 1.
$$

To solve (54) with (55) as an initial value problem we must need the values for $y_3(0)$ i.e. $s''(0)$, $y_5(0)$ i.e. $\theta'(0)$ and $y_7(0)$ i.e. $f'(0)$ but no such values are given. The initial guess values for $s''(0)$, $\theta'(0)$ and $f'(0)$ are chosen and the fourth order Runge–Kutta integration scheme is applied to obtain the solution. Then we compare the calculated values of $s'(\eta)\theta(\eta)$ and $f(\eta)$ at $\eta_\infty$ with the given boundary conditions $s'(\eta_\infty) = 0$, $\theta(\eta_\infty) = 0$ and $f(\eta_\infty) = 0$ and adjust the values of $s''(0)$, $\theta'(0)$ and $f'(0)$ using the shooting technique to give better approximation for the solution. The process is repeated until we get the results correct up to the desired accuracy of $10^{-8}$ level, which fulfils the convergence criterion.

6. Results and discussion

The effects of the pertinent physical parameters on the velocity, temperature and concentration profiles of the nanofluid can be observed from the graphical illustrations (Figs. 2 and 3). In order to verify the present analytical and numerical results, a comparison has been made with Bejan [37] and observed a good agreement which is shown in Table 1. The general trend observed from Figs. 2 and 3 is that the velocity profile is zero at the wall, suddenly increases to a maximum and then it falls down to zero as the distance increases from the wall. It is observed that the
temperature distribution is unity at the wall, and with the change in physical parameters it tends asymptotically to zero in the free stream region. Similar kind of behavior has also been observed for nanosolid volume fraction profiles.

The combined effect of magnetic parameter and Eckert number on non-dimensional velocity, temperature and nanosolid volume fraction profiles of nanofluid are shown in Figs. 2(a)–2(c). It is observed from these Figs, the increasing values of magnetic parameter decrease both velocity and concentration profiles and increase the temperature profile. This is due to the fact that, when a transverse magnetic field is applied to an electrically conducting fluid, it gives rise a resistive force, known as the Lorentz force. This force makes the fluid to experience a resistance
Fig. 2(c). Effects of magnetic parameter and Eckert number on the dimensionless nanosolid fraction profile for $Pr = 4$, $Le = 12$, $\lambda = 0.1$, $N = 0.5$ and $Nb = Nt = Nr = 0.1$.

Fig. 3(a). Effects of heat generation/absorption, Brownian motion and thermophoresis parameters on the dimensionless velocity profile for $Pr = 4$, $Le = 12$, $M = N = 0.5$, $Ec = 1$ and $Nr = 0.1$.

by increasing the friction between its layers and due to which there is a decrease in velocity and concentration. The temperature increases in the boundary layer due to the Ohmic dissipation effect. The increasing values of Eckert number accelerate both the velocity and temperature of nanofluid and decelerate the nanosolid volume fraction profile. This is because, the viscous dissipation produces heat due to drag between the nanofluid particles and this extra heat causes an increase of the initial fluid temperature. The presence of viscous dissipation leads to increase the momentum and thermal boundary layer and decrease the thickness of the concentration boundary layer.
Fig. 3(b). Effects of heat generation/absorption, Brownian motion and thermophoresis parameters on the dimensionless temperature profile for $Pr = 4$, $Le = 12$, $M = N = 0.5$, $Ec = 1$ and $Nr = 0.1$.

Fig. 3(c). Effects of heat generation/absorption, Brownian motion and thermophoresis parameters on the dimensionless nanosolid fraction profile for $Pr = 4$, $Le = 12$, $M = N = 0.5$, $Ec = 1$ and $Nr = 0.1$.

Figs. 3(a)–3(c) show the effect of internal heat generation/absorption, Brownian motion and thermophoresis parameters on the velocity, temperature and solid volume fraction profiles of nanofluid. It can be seen that from these figures, the increasing values of Brownian motion and thermophoresis parameters increase both the velocity and temperature profiles and decrease the nanosolid volume fraction profile. A good variation also can be seen in these figures because of the heat generation ($\lambda > 0$)/absorption ($\lambda < 0$) parameter. The velocity and temperature profiles increase in the case of heat generation and decrease in the heat absorption case. The nanofluid solid volume fraction
Table 1
Comparison test results. Values of the reduced Nusslet number $Nur = Ra_t^{1/4}Nu$ in the limiting case of a regular fluid. The present results are with $Le = 10$, $Nr = Nb = Nt = 10^{-5}$.

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>Bejan [37]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>0.401</td>
<td>0.40103</td>
</tr>
<tr>
<td>10</td>
<td>0.465</td>
<td>0.46496</td>
</tr>
<tr>
<td>100</td>
<td>0.490</td>
<td>0.49000</td>
</tr>
</tbody>
</table>

Fig. 4(a). Variations of $-\theta'(0)$ with $M$ when $Pr = 4$, $\lambda = 0.1$, $Ec = 1$ and $Le = 12$.

profile enhances in heat absorption case and reduces in heat generation case. Due to the fact that the presence of heat generation enhances the momentum and thermal boundary layers thicknesses and reduces the nanofluid concentration boundary layer thickness.

The effect of magnetic parameter, radiation parameter, Buoyancy ratio parameter, Brownian motion and thermophoresis parameters on reduced Nusselt number and local Sherwood number are shown in the Figs. 4(a) and 4(b) respectively. It is clear that the reduced Nusselt number decreases with the increasing values of $M$, $Nb$ and $Nt$ and increases with $Nr$ and $N$ parameters. The same parameters show a opposite effect on local Sherwood number except for higher values of radiation parameter. The combined effect of magnetic parameter with higher values of radiation parameter leads to decrease the values of $-f'(0)$.

Figs. 5(a) and 5(b) show the effect of Eckert number, Prandtl number and heat generation/absorption parameter on reduced Nusselt number and local Sherwood number. It is noted that the increasing values of $Ec$ and $Pr$ decrease the values of $-\theta'(0)$ and increase the values of $-f'(0)$. The reduced Nusselt number increases in heat absorption case and decreases in heat generation case. The heat generation/absorption parameter has an opposite effect on local Sherwood number.

7. Conclusion

A two dimensional steady MHD laminar incompressible boundary layer flow of an electrically conducting nanofluid past a vertical plate with thermal radiation effect in the presence of internal heat generation/absorption, viscous and ohmic dissipations is studied both analytically and numerically. Homotopy analysis method is employed...
to obtain the analytical solution and the numerical solutions are found using fourth order Runge–Kutta method with shooting technique. The specific conclusions obtained from the present study are as follows:

- The nanofluid velocity profile enhances in the presence of viscous dissipation, heat generation and reduces with magnetic field. The nanofluid temperature profile augments in the presence of viscous dissipation, heat generation and magnetic field. The presence of above physical effects shows an opposite effect of the nanosolid volume fraction profile.
- The reduced Nusselt number decreases with magnetic parameter, Eckert number, Prandtl number, heat generation, Brownian motion and thermophoresis parameters and increases with buoyancy ratio and radiation parameters.

Please cite this article in press as: Ganga B, et al. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society (2015), http://dx.doi.org/10.1016/j.jnnms.2015.04.001
local Sherwood has an opposite trend with above said parameters except radiation parameter. The combined effect of magnetic parameter with higher values of radiation parameter leads to decrease the values of local Sherwood number.

Acknowledgments

The authors wish to express their sincere thanks to the editor and the honorable referees for their valuable comments and suggestions to improve the quality of the paper.

References


Please cite this article in press as: Ganga B, et al. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society (2015), http://dx.doi.org/10.1016/j.jnms.2015.04.001

Please cite this article in press as: Ganga B, et al. MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. Journal of the Nigerian Mathematical Society (2015), http://dx.doi.org/10.1016/j.jnms.2015.04.001