Viscoelastic Modeling of Yarn for Tufting Carpet

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Abstract

The viscoelastic properties of polypropylene yarn, used to analyze the stress-strain behavior are characterized in this study. Since the yarn is subjected to dynamic tension and friction during the tufting process, the various mechanical properties are important parameters. In this article, the normal linear solid model and the nonlinear model consisting of spring and dashpot were presented to model the tensile and stress-relaxation response of polypropylene carpet yarn. Then, the experimental data were fitted with the two models to obtain the parameters by applying least square fitting. The result suggests that the standard linear model provided the best fit for the experimental data of the tensile and stress-strain behavior of yarn. The material model can be employed into analysis of vibration characteristic of yarn in tufting carpet process.

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1 Introduction

Studies of the tensile and stress relaxation behavior of viscoelastic carpet yarn have an important role due to the fact that the mechanical properties of yarn have a major effect on the processing and end-use performances of yarns, carpets, such as snarling of yarn and the wrinkle recovery of carpet. Vangheluwe and Kiekens [1] considered that problem of carpet resilience depends on yarn properties. Tatsuo and Yasuo [2] investigated the dynamic behavior (i.e. displacement, stress, strain, etc.) of a filament, which is pulled impulsively at constant velocity, in which three-element viscoelastic model was adopted as the dynamical model of the filament. There are many methods for analysis the viscoelastic properties of textile material. For example, Chailleux and Davies [3, 4] investigated the nonlinear viscoelastic behavior of polyester fiber and aramid fiber based on an analysis of the deformation mechanisms of these materials by applying the macroscopic analysis. Also other authors have focused on studying the viscoelastic behavior of textile material with models consisting of some suitable combination of springs and viscous

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dashpots [5]. The viscoelastic properties of textile material are different than them of elastic solid and viscous fluid. As far as Maxwell model is concerned, the creep compliance is \( \varepsilon(t) = \frac{\sigma}{E} (1 + \frac{1}{\eta} t) \).

Under constant stress, the deformation will be infinite and unrecovered. However, the relaxation modulus of Kelvin model equal to \( \sigma / \varepsilon_c = E \), so the model is Hookean body and has no relaxation effect under constant strain. Above said two models cannot fit stress relaxation and creep behavior of yarn. But, a system of springs and dashpots, which represent elastic solid and fluid-like behavior, may be used to simulate the viscoelastic behavior of polypropylene yarn. Schaff and Ogale [6] investigated the tensile viscoelastic properties of a non-woven polypropylene backing during the dyeing and drying processes using viscoelastic models. Nachane and Sundaram [7] analyzed the stress relaxation and inverse relaxation phenomena of staple fiber by applying linear viscoelastic model. Vangheluwe[8] described the effect of tensile strain rate on ring spun yarn and rotor spun yarn with three parameters nonlinear model and obtained that a normal nonlinear solid model can simulate very well the tensile property of spun yarn. Liu et al., [9]acquired the relaxation modulus of a single spun yarn employing the generalized Maxwell model. A three-element viscoelastic model was presented [10] characterizing the dynamic mechanical properties of polyphenylene fibers used in bulletproof fabric armour. Asayesh and Jeddi [11, 12] studied the creep and fatigue behavior of plain woven fabric respectively, using the yarn creep and fatigue properties, as well as the structural-mechanical parameters of the fabric. The Kelvin model in series with a spring and the Eyring model were developed in their paper as a theoretical analysis to predict the fabric elongation while subjected to a constant load and a cyclic load. The extended nonlinear Maxwell model which based on fitting the parameters to experimental data was demonstrated in literature [13], it was used to calculate relaxation behavior after dynamic loading.

The aim of this study was to investigate the mechanical performance of carpet yarns. Compared to other yarns, however, a carpet yarn generally has relatively more filaments, high strength, high modulus, and high elastic recovery rates. Moreover the yarns were subjected to dynamic tension in the tuft formation. It is important for carpet production to know the effect of the viscoelastic properties of yarns. Therefore, it is worth investigating whether the mechanical behavior of carpet yarn can be simulated by mechanical model. For this purpose, two viscoelastic models which containing various parameters to characterize the tensile and stress relaxation behavior of carpet yarn are established to describe the observed behavior. To verify the validating of the models, the obtained experimental values were fitted to the two models to acquire all the parameters. By characterizing the tensile viscoelastic properties of the yarn and substituting it into the transversal vibration [14], the tension variation of yarn during the tufting process can be better understood and possibly controlled.

2. Mechanical Modulus

In this paper, the first model selected was the standard linear model (a Kelvin model in series with a spring) which has three adjustable parameters. The second model incorporates a nonlinear spring with Maxwell model as shown in Figure 1. The models consist of a series of springs and dashpots which represent the elastic component and viscous component, respectively. In Fig. 1, \( E_1 \), \( E_2 \) and \( E \) are the elastic spring constant (cN/ tex\(^{-1}\)), which is linear with the deformation, and \( \eta \) is the viscosity of the Newtonian piston (cN.s.t tex\(^{-1}\)). In Fig. 1 (b), the model consists of a nonlinear spring, using the following equation between stress \( \sigma \) and strain \( \varepsilon \).

\[
\sigma = b \varepsilon^2
\]  

(1)

Where \( \sigma \) and \( \varepsilon \) are the stress and strain of the spring and \( b \) is the spring constant (cN/ tex\(^{-1}\)) of the nonlinear spring.

The differential Equations (2) and (3) below govern the constitutive relations between stresses \( \sigma \) (cN/tex), strain \( \varepsilon \) (%):
When carpet yarn is elongated at a constant rate, the strain \( \varepsilon \) would be proportional to \( t \), i.e. \( \varepsilon = kt \).

For convenience, we assume that the yarn was clamped distance of 100mm at a speed of 100mm/min:

\[
\frac{k = \frac{v}{l}}{1} = 1, \quad \varepsilon = t
\]  

Substitution of Equation (4) into Equations (2) and (3) leads to the stress-strain equations, solving Equation (2) and (3) with initial conditions \( t = 0 \), \( \sigma_0 = 0 \) give equations of tensile curve.

Model (a):

\[
\sigma = \frac{E_1 E_2 (E_1 + E_2) - \eta^2 E_2}{(E_1 + E_2)^2} (1 - e^{-\frac{E_1 + E_2}{\eta}}) + \frac{E_2 \eta}{E_1 + E_2} \varepsilon
\]  

Model (b):

\[
\sigma = \eta (1 - e^{-\frac{E_2}{\eta}}) + b \varepsilon^2
\]

For stress relaxation, assuming strain is constant \( \varepsilon_c = \varepsilon (s) = k / s \). We can obtain stress equation (7) and (8) from Equation (2) and (3) by Laplace's transformation.

Model (a):

\[
\sigma(t) = \frac{E_1 E_2}{E_1 + E_2} \varepsilon_c (1 + \frac{E_2}{E_1} e^{-\tau / \tau}) , \quad \tau = \eta / (E_1 + E_2)
\]  

Model (b):

\[
\sigma(t) = b \varepsilon_c^2 + E \varepsilon_c e^{-\frac{E_2}{\eta}}
\]

3. Experiment

We tested tensile property of polypropylene carpet yarn with linear density of 1288D using an Instron Universal Testing Machine. Yarn mechanical tests were carried out on 10 samples of 100mm clamp distance at a speed of 100mm/min and the sampling rate was ten points per second under standard laboratory conditions (20°C, 65% relative humidity).
Stress relaxation tests were also conducted with the same Instron tester, at a constant strain of 15%, pre-tension of 2N in the linear viscoelastic range using ten samples allowed to relax for at least 6 minutes.

4. Results and Discussion

The typical representative tensile and stress relaxation curves of the polypropylene yarn from ten tests are displayed in Figure 2 and 3.

In this paper, three-parameter viscoelastic models were used to characterize the mechanical behaviors of yarn. For extracting the discrete viscoelastic parameters, the experimental data collected were fitted to the four equations by applying least square.

It is apparent from Equation (5) and (6) that stress is a function of strain, with parameters $E_1$, $E_2$ and $\eta$, which can be obtained by fitting the experimental data (Figure 4, 5) to Equation (5) and (6) using least squares (see Table 1 and 2). Figure 4 shows the experimental results and fitted curve for the tensile behavior of the yarn by fitting the standard linear and standard nonlinear viscoelastic models. Figure 5 shows similar information, which for the stress relaxation of yarn.

![Stress-strain curve of yarn strand](image1)
![Stress relaxation curve of yarn strand](image2)

**Fig.2.** Dynamic stress-strain curve for polypropylene yarn from tensile tests

**Fig.3.** Stress relaxation curve of polypropylene yarn for carpet

![Fitted curves vs. experimental data for yarn tensile properties](image3)

(a) Standard linear model; (b) Standard nonlinear model

*Fig.4: Fitted curves vs. experimental data for yarn tensile properties.*
As can be seen from Figure 4(a), there was a reasonably good agreement between the fitted curve and the experimental data for the standard linear model, what Figure 4 (b) shows that the standard nonlinear model did not fit the experimental data.

The fitted curves for the tensile data were represented by the following two equations:

Model (a): \[ \sigma = 7.64(1 - e^{-0.12\varepsilon}) + 0.73\varepsilon \] (9)
Model (b): \[ \sigma = 7.67(1 - e^{-0.34\varepsilon}) + 0.02\varepsilon^2 \] (10)

Figure 4(a) shows that the three-element standard linear viscoelastic model provided a reasonable representation of the deformation mechanism for the polypropylene carpet yarn.

Using a typical stress relaxation curve obtained from experimental data, the three-element standard linear and nonlinear models was fitted the stress relaxation data of the yarn as illustrated in Figure 5.

![Figure 5: Fitted curves vs. experimental data for stress relaxation.](image)

(a) Standard linear model; (b) Standard nonlinear model

The respectively stress relaxation fitted curves are given the following two equations:

Model (a): \[ \sigma(t) = 0.91\varepsilon_c(1 + 0.6e^{-0.01t}) \] (11)
Model (b): \[ \sigma(t) = 0.06\varepsilon_c^2 + 0.55\varepsilon_c e^{-0.014t} \] (12)

Form Figure 5, it can be reveal that the three-element standard linear and nonlinear viscoelastic models provided a reasonable representation of the stress relaxation of the polypropylene carpet yarn.

Table 1 Estimated tensile parameters of the polypropylene yarn

<table>
<thead>
<tr>
<th>Model</th>
<th>( E_1/\text{cN.tex}^{-1} )</th>
<th>( E_2/\text{cN.tex}^{-1} )</th>
<th>( \eta/\text{cN.s.tex}^{-1} )</th>
<th>( \tau/\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1.98</td>
<td>0.09</td>
<td>5.32</td>
<td>2.57</td>
</tr>
<tr>
<td>(b)</td>
<td>2.63</td>
<td>0.02</td>
<td>7.67</td>
<td>2.92</td>
</tr>
</tbody>
</table>

Table 2 Estimated stress relaxation parameters of the yarn

<table>
<thead>
<tr>
<th>Model</th>
<th>( E_1/\text{cN.tex}^{-1} )</th>
<th>( E_2/\text{cN.tex}^{-1} )</th>
<th>( \eta/\text{cN.s.tex}^{-1} )</th>
<th>( \tau/\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>2.42</td>
<td>1.46</td>
<td>281.08</td>
<td>72.51</td>
</tr>
<tr>
<td>(b)</td>
<td>0.55</td>
<td>0.06</td>
<td>39.73</td>
<td>72.24</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper, the mechanical properties of polypropylene yarn were evaluated through semi-empirical means. Standard linear and nonlinear three-parameter discrete viscoelastic models were applied to the
tensile and stress relaxation test results, and to simulate the mechanical property as well as to fit the mechanical data predict the model parameters.

According to the analysis, it was possible to conclude that:
The mechanical behavior of the yarn could be represented by the standard linear viscoelastic model using the least squares method to obtain the parameters of the model and to fit the mechanical model to the experimental data.

The good agreement between the fitting and the experimental data suggests that standard models can be used to predict the viscoelastic properties of carpet yarn. It was found that the standard nonlinear viscoelastic model can be only used to predict the stress relaxation behavior and not the tensile behavior of the yarn. The curve derived from nonlinear viscoelastic model did not fit the experimental data.

The parameterized standard linear constitutive models can be used in numerical simulations of vibration characteristic and stress analysis of yarn in tufting process, and these gave predictions that displayed close agreement with actual tension tests. The model which was validated in this study can be used in investigating the vibration of yarn and other dynamics problems associated with the tufting process.

References