

# Dark energy, inflation and the cosmic coincidence problem

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Received 28 June 2007; received in revised form 28 November 2007; accepted 5 February 2008

Available online 7 February 2008

Editor: L. Alvarez-Gaumé

## Abstract

We show that holographic dark energy could explain why the current dark energy density is so small, if there was an inflation with a sufficient expansion in the early universe. It is also suggested that an inflation with the number of e-folds  $N \simeq 65$  may solve the cosmic coincidence problem in this context. Assuming the inflation and the power-law acceleration phase today we obtain approximate formulas for the event horizon size of the universe and dark energy density as functions of time. A simple numerical study exploiting the formula well reproduces the observed evolution of dark energy. This nontrivial match between the theory and the observational data supports both inflation and holographic dark energy models. © 2008 Elsevier B.V. All rights reserved.

PACS: 98.80.Cq; 98.80.Es; 03.65.Ud

The type Ia supernova (SN Ia) observations [1,2] strongly suggest that the current universe is in an accelerating phase, which can be explained by dark energy (a generalization of the cosmological constant) having pressure  $p_\Lambda$  and density  $\rho_\Lambda$  such that  $\omega_\Lambda \equiv p_\Lambda/\rho_\Lambda < -1/3$ . There are various dark energy models rely on exotic materials such as quintessence [3,4],  $k$ -essence [5, 6], phantom [7], and Chaplygin gas [8,9]. Being one of the most important unsolved puzzles in modern physics, the cosmological constant problem consists of three sub-problems; why the cosmological constant is so small, nonzero, and comparable to the critical density at the present.

In this Letter we show that, in the holographic dark energy model, an inflation with a sufficient expansion explain why the current dark energy density is so small. We also suggest that the last problem, the cosmic coincidence problem, could be solved, if there was an inflation with a specific expansion. Note that, in many other dark energy models, it is not easy to explain the current ratio of dark energy density to matter energy density, because usually dark energy density and matter energy density reduce at different rates [10] for a long cosmological time scale.

It is well known [11] that a simple combination of the reduced Planck mass  $M_P = m_P/\sqrt{8\pi}$  and the Hubble parameter  $H = H_0 \sim 10^{-33}$  eV, gives a value  $\rho_\Lambda \simeq M_P^2 H_0^2$  comparable to the observed dark energy density  $\sim 10^{-10}$  eV<sup>4</sup> [2]. This interesting coincidence, on one hand, is of the cosmic coincidence problem and, on the other hand, motivated holographic dark energy models. The holographic dark energy models are based on the holographic principle proposed by 't Hooft and Susskind [12–14], claiming that all of the information in a volume can be described by the physics at the boundary of the volume. With the base on the principle, Cohen et al. [15] proposed a relation between an UV cutoff ( $a$ ) and an IR cutoff ( $L$ ) by considering that the total energy in a region of size  $L$  cannot be larger than the mass of a black hole of that size. Saturating the bound, one can obtain

$$\rho_\Lambda = \frac{3d^2}{L^2 a^2}, \quad (1)$$

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where  $d$  is a constant. Hsu [16] pointed out that for  $L = H^{-1}$ , the holographic dark energy behaves like matter rather than dark energy. Many attempts [17–22] have been made to overcome this IR cutoff problem, for example, by using non-minimal coupling to a scalar field [20,21] or an interaction between dark energy and dark matter [22–26]. Li [27,28] suggested that an ansatz for the holographic dark energy density

$$\rho_\Lambda = \frac{3d^2 M_P^2}{R_h^2}, \quad (2)$$

would give a correct accelerating universe, where the future event horizon ( $R_h$ ) is used instead of the Hubble horizon as the IR cutoff  $L$ .

To solve the coincidence problem many attempts have been done [24,29–34]. An interaction of dark matter [35] with dark energy was introduced in [23,36,37]. In [32] inflation at the GUT scale with the minimal number of e-folds  $N \simeq 60$  was suggested as a solution. In this Letter we suggest a solution similar to the later. One motivation to study the cosmic coincidence problem in the context of inflationary cosmology is that if there was no inflation, there could be no ‘now’ ( $t_0 = 1.37 \times 10^{10}$  years) for the ‘why now’ question. According to astronomical observations and cosmological theory there are at least two inflationary periods in the history of the universe. As is well known, the first inflation at the early universe with  $N > 60$  is need to solve the problems of the standard big-bang cosmology. This inflation is often assumed to be related to vacuum energy of a scalar field (inflaton). The second inflation (re-inflation) is a period of an accelerated expansion today due to dark energy. (Usually, the first inflation is related to a phase transition of the inflaton and has a different origin from that of the re-inflation due to dark energy. In this Letter we assume this case.) Thus, we assume that in the universe there are the inflaton, holographic dark energy, radiation and matter (mostly, cold dark matter). We also assume that after reheating inflaton energy decays to radiation perfectly. During the first inflation holographic dark energy is diluted exponentially. In this work we suggest that if there is holographic dark energy in the universe, the first inflation with  $N \simeq 65$  leads to onset of the second inflation at the time  $t_a = O(10^9)$  years as observed, and, hence, the inflation solves the cosmic coincidence problem in the context of holographic dark energy.

In this Letter we consider the flat ( $k = 0$ ) Friedmann universe which is favored by observations [38] and described by the metric

$$ds^2 = -dt^2 + R^2(t) d\Omega^2, \quad (3)$$

where  $R(t)$  is the scale factor. In the holographic dark energy model a typical length scale of the system with the horizon is given by the future event horizon

$$R_h \equiv R(t) \int_t^\infty \frac{dR(t')}{H(t')R(t')^2} = R(t) \int_t^\infty \frac{dt'}{R(t')}, \quad (4)$$

which is a key quantity. It is a subtle task to obtain an explicit form for  $R_h(t)$ , because  $R_h(t)$  depends on the whole history of the universe after  $t$ . To tackle this problem we divide the history of the universe into two phases; the inflation (phase 1) is followed by phase 2 which are consecutive radiation dominated era (RDE;  $R(t) \propto t^{1/2}$ ) and dark energy dominated power-law accelerating era (DDE;  $R(t) \propto t^n$ ,  $n > 1$ ), respectively. For simplicity, we ignore the matter dominated era (MDE) as often done in an order of magnitude estimate in cosmology. (In Appendix A, we perform a similar calculation with MDE. The main results are similar.)

1) inflation phase ( $t_i \leq t < t_f$ ).

The inflation starts at  $t = t_i$  and ends at  $t_f$ . The scale factor evolves in this phase as follows

$$R(t) = R_i e^{H_i(t-t_i)}, \quad (5)$$

where  $R_i$  is the initial scale factor at  $t = t_i$  and  $H_i = M_i^2/(\sqrt{3}M_P)$  is the Hubble parameter with the energy scale  $M_i$  of the inflation. Hence, the number of e-folds of expansion  $N \equiv H_i(t_f - t_i)$ .

2) power-law expansion phase ( $t_f \leq t < \infty$ ).

This phase consists of RDE ( $t_f \leq t < t_a$ ) followed by DDE ( $t_a \leq t < \infty$ ). The universe starts to accelerate at an inflection point  $t = t_a$ , i.e.,  $\dot{R}(t_a) = 0$ . We assume that the scale factor evolves in this phase as

$$R(t) = R_i e^N \left( \frac{t}{t_f} \right)^{\frac{1}{2}} \left( \frac{1 + \alpha \left( \frac{t}{t_f} \right)^{1/2}}{1 + \alpha} \right)^{2n}, \quad (6)$$

where  $\alpha \simeq (t_f/t_a)^{1/2}$  is a constant. The scale factor  $R(t)$  grows as  $t^{\frac{1}{2}}$  during the RDE and as  $t^{n+\frac{1}{2}}$  during the DDE later.  $R(t)$  of this form gives a smooth transition from RDE to DDE. Note that  $R(t)$  for each era is well known and can be derived from the Friedmann equation depending on the dominant energy source. The power-law acceleration is a generic feature of DDE if  $d > 1$ . (Alternatively, one can divide this phase into RDE and DDE and choose the scale factor as  $R(t) \propto (t/t_f)^{1/2}$  and  $R(t) \propto (t/t_a)^n$  for RDE and DDE, respectively. This choice gives almost the same results except for a slightly decreasing  $R_h$  as  $t \rightarrow t_a$ . Thus, we can use the specific form in Eq. (6) without loss of generality.) Since observational data favor  $d \simeq 1$  [39,40] and  $\omega_\Lambda$  close to

−1, the power index  $n = (1 + d)/(2d - 2)$  is much larger than 1. If we choose  $d = 1.0513$ , then  $n = 20$ . The inflection point  $t_a$  is determined by the value at which the second derivative of  $R(t)$  vanishes:

$$\frac{t_a}{t_f} = \frac{1}{\alpha^2} \left( \frac{\sqrt{5n^2 - 2n} - (n - 1)}{4n^2 - 1} \right)^2. \tag{7}$$

From  $R(t)$  we obtain  $R_h(t)$  using Eq. (4). During the inflationary phase (phase 1):

$$I_1(t) \equiv \int_t^{t_f} \frac{dt'}{R(t')} + \int_{t_f}^{\infty} \frac{dt'}{R(t')} = \frac{e^{-H_i(t-t_i)} - e^{-N}}{H_i R_i} + C(t_f), \tag{8}$$

where  $C(t_f)$  is a constant dependent on  $t_f$ . A finite  $C(t_f)$  implies a finite  $R_h(t)$  and, hence, the existence of DDE. This constant should be determined by the initial condition at  $t_i$ . Thus the distance to the future horizon  $R_h(t)$  during the phase 1 is

$$R_h(t) = R(t)I_1(t) = \frac{1}{H_i} + \left( R_i e^N C(t_f) - \frac{1}{H_i} \right) e^{H_i(t-t_f)}. \tag{9}$$

To determine the value of  $C(t_f)$ , we use an initial condition  $R_h(t_i)$  for  $R_h$ :

$$R_h(t_i) = \frac{1}{H_i} + \left( R_i e^N C(t_f) - \frac{1}{H_i} \right) e^{-N}. \tag{10}$$

Inserting Eq. (10) into Eq. (9) we can rewrite Eq. (9) as

$$R_h(t) = \frac{1}{H_i} (1 + A e^{H_i(t-t_i)}), \tag{11}$$

where  $A$  is a dimensionless constant which depends on the initial condition at  $t = t_i$ , given by

$$A \equiv H_i R_h(t_i) - 1. \tag{12}$$

Therefore, if  $H_i R_h(t_i) > 1$ , i.e.,  $A > 0$ , the event horizon grows exponentially during the inflation. At the same time  $\rho_\Lambda$  decreases exponentially. This is also noted in Ref. [41], where the correction to the inflation due to holographic dark energy was investigated. It is a reasonable assumption that dark energy density at  $t_i$  is comparable to other energy densities, that is,  $\rho_\Lambda(t_i) \sim M_P^2/R_h^2 \sim H_i^2 M_P^2$ , or  $R_h(t_i) \sim H_i^{-1}$ . If not, we need either fine tuning or a special mechanism to make initial dark energy density parameter  $\Omega_\Lambda$  be much smaller than 1 at the extremely early universe, which is implausible. This can be also seen from the following relation [28];

$$H R_h = \frac{d}{\sqrt{\Omega_\Lambda}}, \tag{13}$$

which holographic dark energy model should satisfy all the time. It is  $O(1)$  for  $\Omega_\Lambda$  not too much smaller than 1. Therefore,  $A = O(1)$  is a plausible initial condition.

At  $t = t_f$ ,

$$R_h(t_f) = \frac{1}{H_i} (1 + A e^N). \tag{14}$$

Now consider the phase 2. Using  $R(t)$  in Eq. (6), it is straightforward to obtain the following relations,

$$I_2(t) \equiv \int_t^{\infty} \frac{dt'}{R(t')} = \frac{2}{R_i e^N} \frac{(1 + \alpha)t_f}{(2n - 1)\alpha} \left( \frac{1 + \alpha \sqrt{\frac{t}{t_f}}}{1 + \alpha} \right)^{1-2n}. \tag{15}$$

Therefore, during the phase 2 the event horizon at  $t$  is at the distance

$$R_h(t) = \frac{2t_f}{(2n - 1)\alpha} \left( \sqrt{\frac{t}{t_f}} + \alpha \frac{t}{t_f} \right). \tag{16}$$

Now, the horizon distance at  $t = t_f$  is

$$R_h(t_f) = \frac{2(1 + \alpha)t_f}{(2n - 1)\alpha}. \tag{17}$$

Comparing Eq. (14) with Eq. (17), we have

$$\frac{2(1 + \alpha)t_f}{(2n - 1)\alpha} = \frac{1}{H_i} (1 + A e^N). \tag{18}$$

Therefore, we have

$$\alpha = \left[ \frac{n-1/2}{H_i t_f} (1 + Ae^N) - 1 \right]^{-1} \simeq \frac{H_i t_f}{(n-1/2)Ae^N} \ll 1. \quad (19)$$

Inserting this into Eq. (16) we obtain

$$R_h(t) = \left( \frac{(1 + Ae^N)}{H_i} - \frac{2t_f}{2n-1} \right) \sqrt{\frac{t}{t_f}} + \frac{2t}{2n-1}. \quad (20)$$

Now we have approximate analytical formulas for  $R_h(t)$  for the whole history of the universe since the inflationary era. Note that  $R_h(t)$  is a monotonically increasing function of time. We next consider the behaviors of  $R_h(t)$  for  $t_f < t \ll t_a$ . In this case, we have

$$R_h(t) \simeq t_f \left( \frac{(1 + Ae^N)}{H_i t_f} - \frac{1}{n-1/2} \right) \sqrt{\frac{t}{t_f}} \simeq \frac{Ae^N}{H_i} \sqrt{\frac{t}{t_f}}, \quad (21)$$

which is proportional to  $R(t)$ . From Eq. (7) the inflection point  $t_a$  is given by the initial conditions  $A$  and  $N$  to be

$$\frac{t_a}{t_f} \simeq \frac{(\sqrt{5n^2 - 2n} - n + 1)^2}{4(2n+1)^2} \left( \frac{Ae^N}{H_i t_f} \right)^2 \simeq 0.095 \left( \frac{Ae^N}{N} \right)^2. \quad (22)$$

This relation is interesting and informative. The ratio of the two time scales  $t_a$  and  $t_f$  is related to the initial condition. For  $A > 0$  and  $N \gg 1$ , this ratio explain why dark energy dominates so lately. Eq. (22) implies that  $t_a$  and, hence, evolution of the universe is more sensitive to  $N$  than to  $A$  or  $n$ .

From now on all quantities are given in natural units;  $m_P = 1$ . For  $t_f \simeq 10^7$  (GUT scale inflation) and  $N \simeq 66$ , a reasonable value for inflation to solve the problems of the big-bang cosmology, this equation gives the observed  $t_a \simeq 10^{60}$ . In this way, the holographic dark energy model could solve the cosmic coincidence problem. Interestingly, Eq. (22) gives a lower bound for the energy scale of the inflation. The usual bound  $N \lesssim 60$  for inflation returns  $t_f \lesssim 10^{12}$  and hence  $M_i \lesssim 10^{-7} m_P \sim 10^{12}$  GeV. This can rule out low energy scale inflation models. On the other hand, an obvious condition  $t_f > t_P = 1$  returns  $N < 75$ , where  $t_P$  is the Planck time. To determine the true value of  $N$  we need to go beyond the approximation used in this work.

Let us explain more physically how our model could solve the coincidence problem. Using Eq. (11) we obtain  $R_h(t_f)/R_h(t_i) \simeq e^N$ , which means that the event horizon expands exponentially during the inflation. At the same time the dark energy density  $\rho_\Lambda = 3d^2 M_P^2 / R_h^2$  rapidly decreases (see Fig. 2);

$$\rho_\Lambda(t_f) = \rho_\Lambda(t_i) \left( \frac{R_h(t_i)}{R_h(t_f)} \right)^2 \simeq M_i^4 e^{-2N}, \quad (23)$$

where we used  $\rho_\Lambda(t_i) \simeq M_P^2 H_i^2 \simeq M_i^4$  and  $A \sim O(1)$ . This is the dark energy  $H^2$  density just after the inflation. After the inflation, dark energy is sub-dominant, i.e.,  $\Omega_\Lambda \ll 1$ , and behaves like matter with a constant equation of state [32]

$$\omega_\Lambda = -\frac{1}{3} \left( 1 + \frac{2\sqrt{\Omega_\Lambda}}{d} \right) \simeq -\frac{1}{3}. \quad (24)$$

In this case  $\rho_\Lambda \sim R^{-3(1+\omega)} \sim R^{-2}$ , while the radiation energy density,

$$\rho_r(t) \simeq M_i^4 \left( \frac{R(t_f)}{R(t)} \right)^4, \quad (25)$$

decreases more rapidly than the dark energy density. (From Eq. (21) one can also see that  $R_h(t) \propto R(t) \propto t^{1/2}$  during the RDE). Here we assume an instant reheating after the inflation for simplicity. Therefore, during the RDE

$$\rho_\Lambda(t) \simeq \rho_\Lambda(t_f) \left( \frac{R(t_f)}{R(t)} \right)^2 \simeq \rho_\Lambda(t_f) \left( \frac{t_f}{t} \right) = M_i^4 e^{-2N} \left( \frac{t_f}{t} \right), \quad (26)$$

which should be comparable to  $M_a^4$  at  $t_a$ , where  $M_a \sim 10^{-3}$  eV is the observed energy scale of the universe at the inflection point  $t_a$ . From the above relation, the required e-folds is

$$N \simeq -\frac{1}{2} \ln \left[ \left( \frac{t_a}{t_f} \right) \left( \frac{M_a}{M_i} \right)^4 \right] \simeq \ln \left( \frac{M_i}{M_a} \right) \simeq 64.5, \quad (27)$$

which is slightly larger than the minimal  $N$  for the inflation to solve the many problems of the standard big-bang cosmology. Here we have used  $t_f \sim M_P / M_i^2 \sim M_P / (10^{16} \text{ GeV})^2$ . This result is comparable with heuristic arguments of Li [32,42,43]. Hence, we

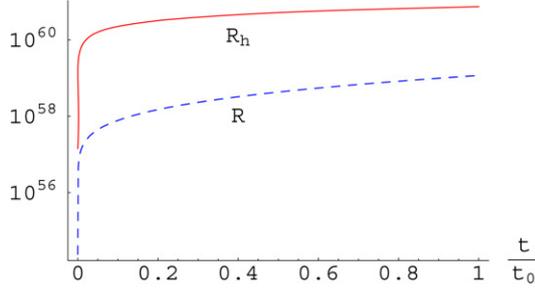


Fig. 1. (Color online.) The size of the event horizon  $R_h$  (red thick line) and the scale factor  $R(t)$  (blue dashed line) as functions of time  $t$  for  $N = 65.7$ ,  $n = 20$ , and  $M_i = 10^{16}$  GeV.  $R_h(t)$  as well as  $R(t)$  grows exponentially during the inflation. All quantities are given in natural units, where  $m_P = 1$ .

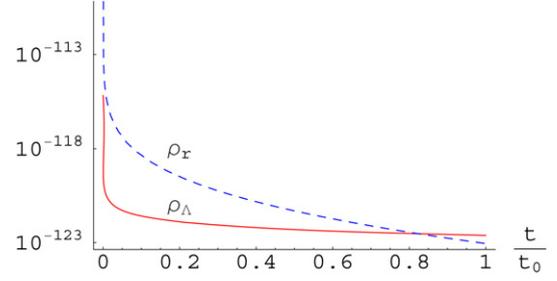


Fig. 2. (Color online.) The dark energy density  $\rho_\Lambda$  (red thick line) and the radiation density  $\rho_r$  (blue dashed line) as a function of time  $t$  for the evolution shown in Fig. 1.  $\rho_\Lambda(t_0) \simeq 2.4 \times 10^{-123}$  is comparable to the observed value. We ignore the matter dominated era for simplicity.

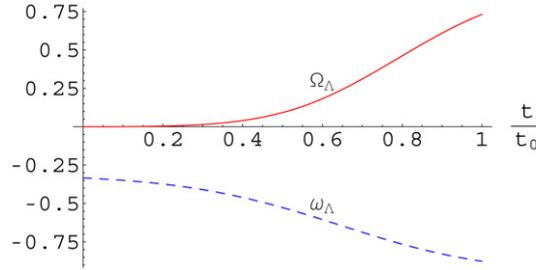


Fig. 3. (Color online.)  $\Omega_\Lambda$  (red thick line) from Eq. (28) and  $\omega_\Lambda$  (blue dashed line) from Eq. (24) of the dark energy as a function of time  $t$  for the evolution shown in Fig. 1.  $\Omega_\Lambda(t_0) = 0.73$  and  $\omega_\Lambda(t_0) = -0.876$  are comparable to the observed values.

see again that an inflation with  $N \simeq 65$  could solve the cosmic coincidence problem in a self-consistent manner in the holographic dark energy context.

To be more concrete we perform a numerical study using the analytic formulas to fit parameters for the inflation such as  $N$  and  $M_i$  onto the observed cosmological parameters such as  $\Omega_\Lambda(t_0)$  and  $\rho_\Lambda(t_0)$ . Once we know  $R_h(t)$  and  $R(t)$ , it is easy to obtain  $\rho_\Lambda(t)$  and  $\rho_r(t)$  by using Eqs. (25) and (26) for a given  $N$ . We choose reasonable values  $M_i = 10^{16}$  GeV,  $A = 1$  and  $n = 20$ . We will show later that our results are not so sensitive to the value of  $A$  or  $n$  as long as  $n \gg 1$ . From Eqs. (21) and (22) one can see that  $t_a$  and

$$\Omega_\Lambda(t) \equiv \frac{\rho_\Lambda(t)}{\rho_\Lambda(t) + \rho_r(t)} \tag{28}$$

are sensitive to  $N$ .

For  $N = 65.7$  our theory gives  $\Omega_\Lambda(t_0) = 0.73$ ,  $\omega_\Lambda = -0.876$ , and  $\rho_\Lambda(t_0) \simeq 2.4 \times 10^{-123}$ , which are comparable with current observations. Note that this is not a fine tuning of  $N$ . Since the expansion during the inflation is a history already happened,  $N$  is obviously a fixed value. Thus,  $N$  value is, like other cosmological parameters, something which should be predicted by a theory and then be verified by observations. One can assert fine-tuning only when a required parameter value is unnatural. Our model *predicts* a value  $N \simeq 65$  which satisfies all known observational constraints and is consistent with inflation theory. Thus, the possibility of determining  $N$  is not necessary a flaw but a possible merit of our theory. Although we cannot rule out  $N \gg 60$ , interestingly, there is an asserted upper bound,  $N \lesssim 65$  from the holographic principle [44–46] and from the density perturbation generation [47,48]. If this upper bound is correct, one can say the holographic dark energy can solve the cosmic coincidence problem.

On the other hand,  $t_a = 0.072t_0$  is smaller than the observed value. This discrepancy can be attributed to approximations we used such as an instant reheating after the inflation and ignoring the matter dominated era. If we choose  $n = 100$  instead of  $n = 20$ ,  $N = 65.715$  gives the same results. Thus, the results are not sensitive to  $n$ . We do not need a fine tuning for  $A$  too. For example, if we choose  $A = 10$ , then we need  $N = 63.39$  to reproduce the observed universe and  $N = 68.01$  for  $A = 10^{-1}$ . As mentioned above a natural value for this dimensionless quantity  $A$  without a fine tuning is  $O(1)$ .

Let us recall the inputs and the outputs in our theory. We have assumed that there are inflationary era, RDE, and DDE in the observed evolution of our universe and used the typical forms of  $R(t)$  for these phases. With reasonable input values for  $n$ ,  $A$  (our results are not sensitive to these values) and  $N \simeq 65$ , we have obtained output values for current density parameter  $\Omega_\Lambda(t_0)$  and equation of state  $\omega_\Lambda(t_0)$  for dark energy, which are comparable with observed values. This could solve the cosmic coincidence problem. Note that assuming DDE without an appropriate inflation does not automatically explain why  $t_a \sim t_0$ . Considering the long time scale involved ( $O(10^{10})$  years) and the difference between time dependency of the dark energy density and that of the matter density, it is remarkable that with the parameter  $N$ , and the reasonable assumptions, our analysis reproduces the observed

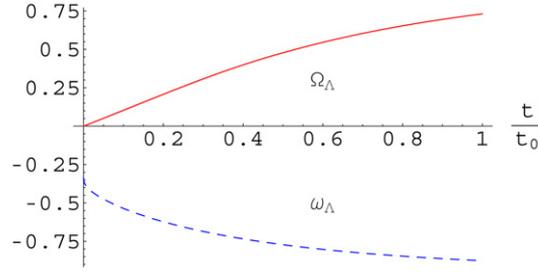


Fig. 4. (Color online.)  $\Omega_\Lambda$  (red thick line) and  $\omega_\Lambda$  (blue dashed line) of the dark energy as a function of time  $t$  for the evolution in Eq. (A.3), which includes the matter dominated era.  $\Omega_\Lambda(t_0) = 0.73$  and  $\omega_\Lambda(t_0) = -0.876$ .

universe with the correct order of magnitude as shown in the figures. This indicates that the holographic dark energy models with  $d \simeq 1$  are promising candidates for a correct dark energy model.

In summary, we show that an inflation with a sufficient expansion make the current holographic dark energy density exponentially small. It is also possible that an inflation of  $N \simeq 65$  could solve the cosmic coincidence problem without introducing an interaction with dark matter or modifying gravity. The holographic dark energy models have an intrinsic advantage over non-holographic models in that it does not need fine tuning of parameters or an *ad hoc* mechanism to cancel the zero-point energy of the vacuum, simply because it has no  $O(M_P^4)$  zero-point vacuum energy from the start. Quantum field theory over-counts the independent physical degrees of freedom inside the volume. Furthermore, as suggested in this Letter, the cosmic coincidence problem could be also solved if there was an inflation with  $N \simeq 65$ . All these results support not only the inflation theory but also the holographic dark energy models with  $d \simeq 1$ .

## Acknowledgements

Authors are thankful to Changbom Park, Yun Soo Myung, Kang Young Lee, and Yeong Gyun Kim for helpful discussions. This work was supported by Daejin University Research Grants in 2007.

## Appendix A. Including MDE

In this appendix, we investigate the effect of matter dominated era (MDE) on the evolution of holographic dark energy. We assume that there are a period of the inflation followed by the radiation dominated era (RDE), a slow transition from matter dominated era to dark energy dominated era (MDE + DDE) of which scale factors are given by

$$R(t) = \begin{cases} R_i e^{H_i(t-t_i)}, & t_i \leq t < t_f \text{ (Inflation)}, \\ R_i e^N \left(\frac{t}{t_f}\right)^{\frac{1}{2}}, & t_f \leq t < t_{\text{eq}} \text{ (RDE)}, \\ R_i e^N \left(\frac{t_{\text{eq}}}{t_f}\right)^{\frac{1}{2}} \left(\frac{t}{t_{\text{eq}}}\right)^{\frac{2}{3}} \left(\frac{1+\alpha(\frac{t}{t_{\text{eq}}})^{1/3}}{1+\alpha}\right)^{3n}, & t_{\text{eq}} \leq t \text{ (MDE + DDE)}, \end{cases} \quad (\text{A.1})$$

respectively, where  $\alpha$  is a constant. We set the transition from the radiation dominated era (RDE) to the matter dominated era (MDE) happens at  $t = t_{\text{eq}}$ , the equipartition time. That is,  $\rho_r(t_{\text{eq}}) = \rho_m(t_{\text{eq}})$ , where  $\rho_m$  is matter energy density. The last phase consists of the matter dominated era (MDE) ( $t_{\text{eq}} \leq t < t_a$ ) followed by a dark energy dominated era (DDE) ( $t_a \leq t < \infty$ ). The inflection point  $t_a$  is determined by the value in which the second derivative of  $R(t)$  vanishes during the third phase:

$$\frac{t_a}{t_{\text{eq}}} \simeq \left(\frac{\sqrt{3}-1}{3\alpha n}\right)^3. \quad (\text{A.2})$$

Using the scale factors in Eq. (A.1), it is easy to derive the event horizon size  $R_h$  and dark energy density  $\rho_\Lambda$  by following the procedure in the main text. After some straightforward calculation we obtain  $R_h$  for each phase;

$$R_h(t) = \begin{cases} \frac{1}{H_i} (1 + A e^{H_i(t-t_i)}), & t_i \leq t < t_f \text{ (Inflation)}, \\ \frac{1 + A e^N + 2H_i t_f}{H_i} \sqrt{\frac{t}{t_f}} - 2t, & t_f \leq t < t_{\text{eq}} \text{ (RDE)}, \\ t_{\text{eq}} \left[ \left( \frac{1 + A e^N + 2H_i t_f}{H_i \sqrt{t_{\text{eq}} t_f}} - 2 - \frac{1}{n-1/3} \right) \left(\frac{t}{t_{\text{eq}}}\right)^{2/3} + \frac{1}{n-1/3} \frac{t}{t_{\text{eq}}} \right], & t_{\text{eq}} \leq t \text{ (MDE + DDE)}. \end{cases} \quad (\text{A.3})$$

During the calculation, from the continuity condition of  $R_h$  between RDE and MDE, we have obtained

$$\alpha = \left[ \frac{1 + A e^N + 2H_i t_f}{H_i \sqrt{t_{\text{eq}} t_f}} - 2 - \frac{1}{n-1/3} \right]^{-1} \frac{1}{n-1/3} \quad (\text{A.4})$$

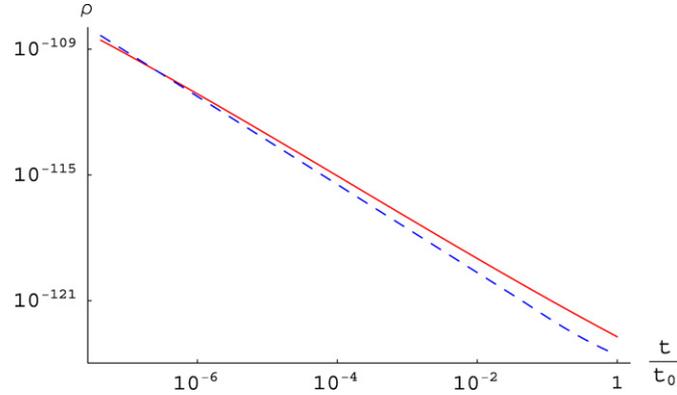


Fig. 5. (Color online.)  $3H^2(t)M_p^2$  (red thick line) and the total energy density  $\rho_\Lambda(t) + \rho_r(t) + \rho_m(t)$  (blue dashed line) as functions of time  $t$  for the evolution in Eq. (A.3).

and used this  $\alpha$  for following calculations. In generic holographic dark energy models, dark matter is independent of dark energy, and we need an parameter describing the nature of dark matter. We choose the observed equipartition time  $t_{\text{eq}} \simeq 10^{-7}t_0$  ( $z_{\text{eq}} \simeq 3200$ ) for the parameter. Fig. 4 shows the results for  $N = 61.9$ .  $N$  becomes smaller compared to the case in Fig. 3, because the matter energy density decreases slowly ( $\rho_m \propto R^{-3}$ ) than the radiation energy density ( $\rho_r \propto R^{-4}$ ). The other parameters are the same as those of Fig. 3. As assumed in the main text, including MDE in our consideration does not significantly change the results. Compared to the case without MDE (Fig. 3),  $\Omega_\Lambda(t) \equiv \rho_\Lambda(t)/(\rho_\Lambda(t) + \rho_r(t) + \rho_m(t))$  curve is more flat and  $t_a \simeq 5 \times 10^9$  years is later. These results are more consistent with observations, while  $\rho_\Lambda(t_0) \simeq 2.3 \times 10^{-124}$  is slightly smaller than the observed value. Since our holographic dark energy density changes about  $10^{107}$  times in scale from the inflation to the present, this level of coincidence is interesting, considering the approximations we have used. To check the accuracy of our calculation using the Friedmann equation, we plot the total energy density and  $3H^2(t)M_p^2$  in Fig. 5. The graph shows the level of accuracy mentioned above.

In the case with MDE considered in this appendix, due to the freedom of  $t_{\text{eq}}$ , there was no guarantee that the inflation with  $N \sim 65$  could solve the cosmic coincidence problem. However, interestingly, it turns out that even in this case the required  $N$  value is similar to that of the case without MDE. This is due to the fact that the observed initial dark matter density is much smaller than that of radiation. Even in the worst case that MDE started just after the reheating of the inflation and there was no RDE, Eq. (26) with  $R \sim t^{2/3}$  gives a value  $N \simeq \frac{2}{3} \ln(\frac{M_i}{M_a}) \sim 43$ .

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