Flexural Strength and Deformability Design of Reinforced Concrete Beams

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Abstract

In the flexural design of reinforced concrete (RC) beams, the strength and deformability, which are interrelated, need to be considered simultaneously. However, in current design codes, design of strength is separated with deformability, and evaluation of deformability is independent of some key parameters, like concrete strength, steel yield strength and confinement content. Hence, provisions in current design codes may not provide sufficient deformability for beams, especially when high-strength concrete (HSC) and/or high-strength steel (HSS) are used. In this paper, influences of key factors, including the degree of reinforcement, concrete strength, steel yield strength, compression steel ratio, and confining pressure, are studied based on a theoretical method. An empirical formula for direct evaluation of deformability is proposed. Interrelation between the strength and deformability are plotted in charts. Based on the empirical formula and charts, a new method of beam design called “concurrent flexural strength and deformability design” that would allow both strength and deformability requirements to be considered simultaneously is developed. The method provides engineers with flexibility of using high-strength concrete, adding compression steel or adding confinement to increase deformability of RC beams.

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1. INTRODUCTION

In the flexural design of reinforced concrete (RC) beams, deformation capacity should be regarded as important as strength. Adequate deformation capacity can ensure structural safety when a structure is overloaded, by dissipating excessive energy through plastic hinges. However, in current design codes, there are only some empirical deemed-to-satisfy rules that limit maximum allowable tension steel area or neutral axis depth. Such provisions can ensure a beam is under-reinforced and has a certain level of deformation capacity, but they are far from sufficient. First, in performance-based design, deformation capacity needs to be quantitatively evaluated and designed; no formula or method that is accurate enough is provided in current codes (ACI Committee 2008; Ministry of Construction 2001; ECS 2004; Standards New Zealand 2006). Second, these empirical rules are developed based on knowledge from beams made of normal strength concrete (NSC) and normal strength steel (NSS). With the increasingly popular use of high strength concrete (HSC) and high strength steel (HSS), these rules, which are commonly independent of concrete and steel grade, may not be able to provide consistent deformation capacity for beams made of HSC and HSS. Third, strength and deformation capacity are interrelated and affected simultaneously by some factors, the interrelation factors needs to be well investigated.

In terms of deformation capacity, flexural ductility has been widely studied and applied. However, the flexural deformability in terms of ultimate rotation capacity is also important from performance-based design point of view (Rubinstein et al. 2007; Challamel 2009). Due to usually higher initial stiffness for beams made of HSC and/or HSS, a beam provided with sufficient ductility may not have sufficient deformability, because ductility is defined as deformation capacity at ultimate state relative to that at the initial yield state.

In this study, the flexural deformability of beam sections is comprehensively studied. The effects of major factors on flexural deformability and strength are investigated. Empirical formula for direct evaluation of flexural deformability and a “concurrent flexural strength and deformability design” method are proposed.

2. NONLINEAR MOMENT-CURVATURE ANALYSIS

A nonlinear moment-curvature analysis developed previously by the authors (Pam et al. 2001; Ho et al. 2003) has been adopted in this study. Details of this method can be found accordingly. The outline of the method is given here:

Five basic assumptions are made in the analysis: (1) Plane sections before bending remain plane after bending. (2) The tensile strength of the concrete may be neglected. (3) There is no relative slip between concrete and steel reinforcement. (4) The concrete core is confined while the concrete cover is unconfined. (5) The confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Park et al. 2007; Au et al. 2009; Lam et al. 2008; Kwak and Kim 2010). Assumption (5) is not exact but fairly reasonable (Ho et al. 2010).

The stress-strain curves of concrete developed by Attard and Setunge (1996) are adopted. For steel reinforcement, the idealised linearly elastic – perfectly plastic stress-strain curve is adopted. At a given curvature, the stress and strain of concrete and steel, and the axial force and moment of the section, depend only on the neutral axis depth, which can be determined by applying axial force and moment equilibrium conditions across the section. Having determined the neutral axis depth, the axial force and moment can be calculated. To obtain a moment-curvature curve of a section, the above procedure is repeated by increasing curvature step by step until the section goes into post-peak stage and the moment drops to less than 50% of the peak moment.
3. EVALUATION OF FLEXURAL DEFORMABILITY

3.1. Flexural deformability
The flexural deformability of beam sections is expressed in terms of normalised rotation capacity $\theta_{pl}$ defined as follows:

$$\theta_{pl} = \phi_u d$$

where $\phi_u$ is the ultimate curvature, $d$ is the effective depth of the beam section. The ultimate curvature $\phi_u$ is taken as the curvature when the resisting moment has dropped to 0.8$M_p$ after reaching $M_p$, where $M_p$ is the peak moment. The value of $\theta_{pl}$ evaluated from equation (1) would give the rotation capacity of the beam with plastic hinge length equal to the effective depth.

3.2. Flexural failure modes
There are three failure modes for beam sections in flexure: (1) Tension failure – tension steel yields during failure; (2) Compression failure – none of tension steel yields during failure; and (3) Balanced failure – the most highly stressed tension steel has just yielded during failure. The tension steel ratio in balanced failure is called balanced steel ratio. From previous studies (Pam et al. 2001; Ho et al. 2003), the balanced steel ratio for single reinforced beam section $\rho_{bo}$ and for doubly reinforced beam section $\rho_{bd}$ can be evaluated by equations (2) and (3) respectively:

$$\rho_{bo} = 0.005 \left( f_{co} \right)^{0.58} \left( 1 + 1.2 f_t \right)^{0.3} (f_{yt} / 460)^{-1.35}$$

$$\rho_{bd} = \rho_{bo} + \left( f_{yc} / f_{yt} \right) \rho_c$$

where $f_{co}$ is the concrete strength, $\rho_c$ is the tension steel ratio, $f_{yt}$ is the tension steel yield strength, $f_{yc}$ is the compression steel yield strength, $f_t$ is the confining pressure evaluated using the method recommended by Mander et al (1988). It was also revealed that the degree of reinforcement $\lambda$ defined by equation (4) is a good indicator of the failure mode. The beam section is classified as under-reinforced, balanced and over-reinforced sections when $\lambda$ is less than, equal to and larger than 1.0 respectively. In equation (4), $f_r$ is the tension steel ratio.

$$\lambda = \frac{f_{yt} \rho_r - f_{yc} \rho_c}{f_{yt} \rho_{bo}}$$

3.3. Factors affecting flexural deformability
A comprehensive parametric study on the effects of major factors on flexural deformability was conducted. The studied factors are: (1) Tension and compression steel ratios expressed in terms of degree of reinforcement; (2) Concrete strength; (3) Steel yield strength; (4) Confining pressure provided by transverse reinforcement. The beam sections analysed is shown in Figure 1.
The effects of tension steel ratio and compression steel ratios on deformability are shown in Figure 2. In Figure 2(a), $\theta_{pl}$ decreases as $\rho_t$ increases until $\rho_t$ reaches balanced steel ratio, after which $\theta_{pl}$ remains relatively constant. At a fixed $\rho_t$, $\theta_{pl}$ increases as $\rho_c$ increases. Hence, reducing tension steel and adding compression steel can improve deformability.

The same trend is expressed with $\lambda$ in Figure 2(b), where the variation of $\lambda$ is due to variation of $\rho_t$. The deformability $\theta_{pl}$ increases until $\lambda$ reaches 1.0, after which $\theta_{pl}$ remains relatively constant. At a fixed $\lambda$, $\theta_{pl}$ increases as $\rho_c$ increases. It is evident that degree of reinforcement $\lambda$, which combines effects of tension and compression steel ratios, can be a good parameter to uniformly express the trend of deformability. To facilitate obtaining a formula for direct evaluation of deformability, the effects of other factors are studied firstly at a fixed $\lambda$, their effects at a fixed $\rho_t$ will be discussed later.

At a fixed $\lambda$, the effects of concrete strength, tension steel yield strength and confining pressure on deformability are shown in Figure 3, Figure 4 and Figure 5 respectively. It is observed that the use of HSC at a fixed $\lambda$ would reduce deformability, the use of HSS as tension steel at a fixed $\lambda$ could improve deformability, and increasing confining pressure can improve deformability significantly.
3.4. Evaluation of flexural deformability

Based on the effects of these factors, a regression analysis is conducted and the following equations are obtained for rapid evaluation of deformability of under-reinforced beam sections ($\lambda < 1.0$):

$$\theta_{pl} = 0.03 m \left( f_{co} \right)^{-0.3} \left( \lambda \right)^{-1.0 n} \left( 1 + 110 \left( f_{co} \right)^{-1.1} \left( \frac{f_{yc}}{f_{yt} \rho_t} \right)^{3} \left( \frac{f_{st}}{460} \right)^{0.3} \right) $$  \hspace{1cm} (5a)

$$m = 1 + 4 \left( f_{co} \right)^{0.4} \left( f_{y} / f_{co} \right) $$  \hspace{1cm} (5b)

$$n = 1 + 3 \left( f_{co} \right)^{0.2} \left( f_{y} / f_{co} \right) $$  \hspace{1cm} (5c)

where $f_{co}, f_{yc}, f_{st}$ and $f_{y}$ are in MPa, $\lambda \leq 1.0$ and $400 \leq f_{yc} \leq 800$ MPa. The validity of the equation has been compared with available experimental results in the authors’ previous study (Zhou et al. 2010). For over-reinforced beam sections ($\lambda > 1.0$), the deformability can be evaluated by replacing $\lambda = 1.0$ in Eq. (5a) since the effect of $\lambda$ on deformability is now insignificant.

Furthermore, due to the complexity of distribution of confining pressure, the positive effects of confining pressure are sometimes considered as safety reserve. For under-reinforced beam sections where confining pressure is not considered and $\rho_c$ is much less than $\rho_t$, equation (5) can be simplified into equation (6).
\[ \theta_{pl} = 0.03(f_{co})^{-0.3}(\lambda)^{-1.0}\left(\frac{f_{yt}}{460}\right)^{0.3} \]  

(6)

Substituting equation (2) and equation (4) into equation (6), the following equation is obtained.

\[ \theta_{pl} = \frac{0.09(f_{co})^{0.28}}{f_{yt}\rho_t - f_{yc}\rho_c} \]  

(7)

From equation (7), it can be concluded that: (1) Reducing tension steel and adding compression steel can improve deformability; (2) The use of HSC at a fixed \( \rho_c \), in other words simply replacing NSC with HSC, could improve deformability, although HSC is relatively more brittle than NSC. (3) When HSS is used as tension steel ratio, as long as the product of \( f_{yt}\rho_t \) is the same, the deformability is the same. This can also be revealed from the comparison of two typical moment-curvature curves shown in Figure 6, where the two sections share the same peak and post-peak curve and hence have the same strength and deformability.

4. INTERRELATION BETWEEN FLEXURAL STRENGTH AND DEFORMABILITY

To study how the above factors influence flexural strength and deformability simultaneously, the deformability is plotted against strength, with tension steel ratio varying.

In Figure 7, curves represent different concrete grades used. In each curve, with tension steel ratio increasing, the deformability drops while the strength increases. This shows the strength is improved at the cost of deformability, or the deformability is improved at the cost of strength. On the other hand, the curves shift upper right from NSC to HSC, indicating the use of HSC can improve the strength while maintaining the same deformability, vice versa. This shows that the limit to strength and deformability that can be simultaneously achieved is extended, which is a major merit of using HSC.

In Figure 8, three curves were produced, each corresponding to a steel grade. However, the curves overlap each other, which means the change of steel grade will not change the limit to strength and deformability that can be simultaneously achieved. This is easily explained by Figure 6 which shows that two sections with the same value of \( f_{yt}\rho_t \) share the same peak and post-peak curve and hence have the same strength and deformability. Therefore, the use of HSS can reduce steel area but can not improve the total performance.

![Figure 7: Effects of concrete strength](image1)

![Figure 8: Effects of tension steel yield strength](image2)
Figure 9 shows the effects of compression steel. Each curve is produced at a fixed amount of compression steel ratio. The descending trend of each curve shows there is a limit to the strength and deformability that can be simultaneously achieved at a fixed amount of compression steel ratio, while the upper-right shift of the curves from low to high compression steel ratio shows that adding compression steel can extend the limit.

Figure 10 shows the effects of confining pressure. Each curve is produced at a fixed amount of confining pressure. It is evident that increasing confining pressure can improve strength and deformability significantly at the same time.

5. CONCURRENT DESIGN OF FLEXURAL STRENGTH AND DEFORMABILITY

With equation (5) and Figure (7), (8) and (9), a design method called “Concurrent strength and deformability design” is proposed. This method gives designers multiple choices to design a beam section with required strength and deformability.

With a given strength and deformability requirement, the beam section should be designed traditionally according to the strength requirement to obtain the required tension steel ratio, and then the deformability can be checked using equation (5). If the deformability requirement is not satisfied, the adjustment of tension steel ratio will not be able to satisfy the strength and deformability requirement at the same time. Designers can choose to use HSC or add compression steel or add transverse steel. Because of the cost of steel and steel congestion problem, it is recommended to first try HSC. If concrete grade is prescribed or using HSC can not provide sufficient deformability, then compression steel or transverse steel is needed. If all these ways fails, then there is no way but to enlarge the section size.

In current design codes, it is a common way to limit the maximum allowable tension steel ratio to ensure a certain level of deformability. However, provisions in current codes are usually independent of designer-specified deformability requirement, or concrete/steel grade or compression steel conditions. To enable rapid check of maximum allowable tension steel ratio under a specific design deformability requirement, equation (8) is derived from equation (7).

\[
\rho_{t,\text{max}} = \left( \frac{0.065 f_{co}^{0.28}}{\theta_{pl,\text{min}}} + f_{yc} \rho_c \right) / f_{yt}
\]

With equation (8), it is more straightforward to make design choices. If the required tension steel ratio according to the strength requirement is larger than \( \rho_{t,\text{max}} \) evaluated from equation (8), then \( \rho_{\text{t, max}} \) can be increased by using HSC or adding compression steel. If HSS is used, it should be noted that the \( \rho_{t,\text{max}} \) is
reversely proportionally reduced. Since the effect of confining pressure is taken as safety reserve, a design that satisfies the requirement of equation (8) provides larger deformability.

6. CONCLUSIONS

Based on the method of non-linear moment-curvature analysis, the effects of major factors, including tension/compression steel ratio, concrete/steel grade, confining pressure, on flexural strength and deformability are studied. It is found that adjusting tension steel can only increase deformability at the cost of strength or increase strength at the cost of deformability. To improve strength and deformability at the same time at a specific section size, designers should choose to use HSC or add compression steel or add transverse steel.

To enable quantitative design of deformability, equations are derived from regression analysis. Based on the equations, a design method called “Concurrent strength and deformability design” is proposed. This method allows designers to consider strength and deformability requirements at the same time, and provides designers with the flexibility to make design choices.

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