Nonlinear Aeroelastic Response of High-aspect-ratio Flexible Wings

Zhang Jian, Xiang Jinwu*

School of Aeronautic Science and Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

Received 28 August 2008; accepted 7 November 2008

Abstract

The aeroelastic analysis of high-altitude, long-endurance (HALE) aircraft that features high-aspect-ratio flexible wings needs take into account structural geometrical nonlinearities and dynamic stall. For a generic nonlinear aeroelastic system, besides the stability boundary, the characteristics of the limit-cycle oscillation (LCO) should also be accurately predicted. In order to conduct nonlinear aeroelastic analysis of high-aspect-ratio flexible wings, a first-order, state-space model is developed by combining a geometrically exact, nonlinear anisotropic beam model with nonlinear ONERA (Edlin) dynamic stall model. The present investigations focus on the initiation and sustaining mechanism of the LCO and the effects of flight speed and drag on aeroelastic behaviors. Numerical results indicate that structural geometrical nonlinearities could lead to the LCO without stall occurring. As flight speed increases, dynamic stall becomes dominant and the LCO increasingly complicated. Drag could be negligible for LCO type, but should be considered to exactly predict the onset speed of flutter or LCO of high-aspect-ratio flexible wings.

Keywords: nonlinear aeroelasticity; limit-cycle oscillation; Galerkin methods; geometrical nonlinearities; dynamic stall; HALE aircraft

1. Introduction

In the aeroelastic analysis of high-aspect-ratio flexible wings of high-altitude, long-endurance (HALE) aircraft, structural geometrical nonlinearities and dynamic stall should be taken into account. The large deformation due to the high flexibility and accompanying changes in aerodynamic loads could produce significant changes in the aeroelastic behavior of wings, and it is very likely that the wing tip will encounter stall when the aircraft flies at a high altitude and a low speed with a large trim angle of attack. Furthermore, besides the flutter boundary, the characteristics of the limit-cycle oscillation (LCO) also should be accurately predicted for a generic nonlinear aeroelastic system since the LCO will cause structure fatigue to reduce the service life of the structure or immediate failure if the amplitude is high enough.

P. Dunn, et al.[1] conducted theoretical and experimental studies on the characteristics of stall flutter and divergence of cantilevered composite laminated wings, and established a simple analytic aeroelastic model by using an empirical cubic torsional stiffness model and ONERA dynamic stall model without drag included. The theoretical and experimental studies conducted by D. Tang, et al.[2] paid attention to the effects of structural geometrical nonlinearities, dynamic stall and steady angle of attack on the flutter instability boundary and the LCO of a high-aspect-ratio wing with a tip slender body. Structural equations of motion were based on a uniform, untwisted beam theory[3] that accounts for small strain and moderate deformation with third and higher order nonlinearity terms neglected. The ONERA dynamic stall model together with a quasi-steady drag equation was used. As a continuation work, limit-cycle hysteresis response was also investigated[4]. M. J. Patil, et al.[5-8] have established a theoretical basis for nonlinear aeroelastic analysis of HALE aircraft that takes into account material anisotropy, structural geometrical nonlinearities, dynamic stall and rigid-body motions. Their investigation gave an insight into the effects of nonlinearities and mutual interactions between the wing flexibility and rigid-body motions on aeroelastic behaviors and flight dynamic characteristics[6,7]. The effects of speed and disturbance on the characteristics of LCO were also investigated[6,8]. The structures were modeled by using the mixed variational formulation based on geome-

drically exact, intrinsic theory for dynamics of moving beams[9]. The aerodynamic model was based on the
finite-state aerodynamic theory complemented with ONERA dynamic stall model\cite{10}. The studies conducted by X. N. Liu, et al.\cite{11-13} addressed the effects of nonlinearities, ply angles, and wing parameters on the aeroelastic behaviors of high-aspect-ratio composite wings taking into account material anisotropy, structural geometrical nonlinearities and dynamic stall. An aeroelastic tailoring optimization method was also provided\cite{11}. The wing was modeled by using linear theory for composite beams under large deflections with Euler assumption\cite{14,15}. The ONERA dynamic stall equations given by P. Dunn, et al.\cite{1} were used without considering the absence of displacement and finite rotation variables from the formulations. Thus, it avoids the need for linearization of the ONERA dynamic stall model \cite{10}. The studies conducted by Zhang Jian et al.\cite{1} were used without considering drag.

In the present study, a geometrically exact, nonlinear anisotropic beam model is combined with a nonlinear dynamic stall model with drag included to develop a first-order, state-space model for nonlinear aeroelastic analysis of high-aspect-ratio flexible wings. It takes into account material anisotropy, structural geometrical nonlinearities and dynamic stall with all nonlinearity terms retained. The present investigations focus on the initiation and sustaining mechanism of the LCO and the effects of flight speed and drag on aeroelastic behaviors.

2. Formulation

2.1. Structural model

In the present study, half a high-aspect-ratio wing is represented as a cantilever beam. A geometrically exact, fully intrinsic theory developed by D. H. Hodges\cite{16} for dynamics of initially curved and twisted anisotropic Timoshenko moving beams is used as the fundamental structural model. This theory models a 1D beam undergoing small strain and large deformation. One of the main advantages of this theory is that the order of nonlinearity is no greater than two due to the absence of displacement and finite rotation variables from the formulations. Thus, it avoids the need for introducing trigonometric or rational functions etc. to represent the finite rotation that would make it intractable to realize exact coupling of the structural model and the aerodynamic model.

The intrinsic equations for nonlinear dynamics of beams are

\[
F' + (\hat{k} + \kappa)F + f = P + \Omega P
\]

\[
M' + (\hat{k} + \kappa)M + (\hat{\xi} + \hat{\gamma})F + m = H + \Omega H + \hat{\gamma}P
\]

\[
V' + (\hat{k} + \kappa)V + (\hat{\xi} + \hat{\gamma})\Omega = \dot{\gamma}
\]

\[
\Omega' + (\hat{k} + \kappa)\Omega = \kappa
\]

where Eqs.(1)-(2) are the equations of motions and Eqs.(3)-(4) the intrinsic kinematical equations; \( \hat{\gamma} = \partial\gamma/\partial t \) and \( \hat{\xi} = \partial\xi/\partial t \) denote the derivatives with respect to absolute time and the undeformed beam reference line respectively, \( ' \) denotes a cross product operator applied to vectors; \( F \) and \( M \), called generalized forces, are the internal force and moment measurements; \( \gamma \) and \( \kappa \), called generalized strains, are the beam strains and curvatures; \( P \) and \( H \), called generalized momenta, are the sectional linear and angular momenta; \( V \) and \( \Omega \), called generalized velocities, are the inertial and angular velocities; \( f \) and \( m \) are the external distributed force and moment per unit length. All the variables are expressed in the cross-sectional reference frame of undeformed beam. In addition, \( k = [k_1, k_2, k_3]^T \) defines the initial twist and curvature of the beam, \( e = [1, 0, 0]^T \).

Generalized strains and velocities are linearly linked to generalized forces and momenta by the cross-sectional constitutive laws:

\[
\begin{bmatrix}
F \\
M \\
\end{bmatrix} = \begin{bmatrix}
X & Y \\
Y^T & Z
\end{bmatrix} \begin{bmatrix}
\gamma \\
\kappa
\end{bmatrix}
\]

\[
\begin{bmatrix}
P \\
H
\end{bmatrix} = \begin{bmatrix}
G & K \\
K^T & I
\end{bmatrix} \begin{bmatrix}
V \\
\Omega
\end{bmatrix}
\]

The cross-section stiffness matrix in Eq.(5) for an arbitrary closed cross section can be obtained through variational asymptotic beam sectional analysis\cite{17}. The inertia matrices in Eq.(6) have the following components:

\[
G = \mu A
\]

\[
K = -\mu \bar{\xi}
\]

\[
I = \begin{bmatrix}
i_1 + i_3 & 0 & 0 \\
0 & i_2 & i_3 \\
0 & i_3 & i_3
\end{bmatrix}
\]

where \( A \) is the identity matrix, \( \mu \) the mass per unit length, \( \bar{\xi} = [0 \bar{x}_3 \bar{x}_3]^T \) the offset vector of mass center, \( i_1, i_2, i_3 \) are the cross-sectional mass moments and product of inertia.

In the present aeroelastic analysis, \( V, \Omega, \gamma \) and \( \kappa \) are chosen as primary variables. The external loads include aerodynamic and gravitational ones and are expressed by \( f = f_{aero} + f_{g} \) and \( m = m_{aero} + m_{g} \).

2.2. Semi-discretization of structural model

The structural model is discretized in space using the Galerkin finite-element method\cite{18}. The first step is to formulate the original governing equations in equivalent integral representation called the weak form in finite-element analysis (For convenience, the coordinate variable \( x \) used above is replaced by \( x \)):

\[
\int_0^l \phi' \left[ \begin{array}{c}
P + \Omega P - F' \\
-\hat{k}F - f
\end{array} \right] dx = 0
\]

\[
\int_0^l \phi' \left[ \begin{array}{c}
H + \Omega H + \hat{\gamma}P - M' \\
-\hat{k}M - \hat{\gamma}F - m
\end{array} \right] dx = 0
\]

\[
\int_0^l \phi' \left[ \begin{array}{c}
\dot{\gamma} - V' - (\hat{k} + \kappa) V - (\hat{\xi} + \hat{\gamma}) \Omega \\
\kappa - \Omega - (\hat{k} + \kappa) \Omega
\end{array} \right] dx = 0
\]
ence line. The primary variable can be expressed as a linear combination of the trial functions weighted by nodal values:

$$
V(x,t) = \Phi^I(x)v^I(t),
$$

$$
\Omega(x,t) = \Phi^I(x)\Omega^I(t),
$$

$$
\gamma(x,t) = \Phi^I(x)\gamma^I(t),
$$

$$
\kappa(x,t) = \Phi^I(x)\kappa^I(t)
$$

where $\Phi^I(x) = N(x)A$ with $N(x)$ being the trial function, which is the linear Lagrangian interpolation function. The superscript $I = 1, 2, \cdots, n+1$ denotes the element node index. Einstein’s summation convention is used.

Assume that the cross-sectional stiffness and inertia properties as well as the initial twist and curvature of the beam are constant. Substitute Eqs.(5)-(6) and Eq.(12) into Eqs.(8)-(11) and one can obtain the semi-discrete equations as follows:

$$
A^H(Gv^I + Ko^I) + C_{JK} \tilde{K}(Gv^I + Ko^I) - B^H(\lambda_f + YK^I - A^H \tilde{K}(X_f + YK^I) - 
$$

$$
C_{JK} \tilde{K}(X_f + YK^I) - \int_0^1 \Phi^J f dm dx = 0
$$

$$
A^H(K^T v^I + \dot{w}^I) + C_{JK} \tilde{K}(K^T v^I + \dot{w}^I) +
$$

$$
C_{JK} \tilde{K}(K^T v^I + \dot{w}^I) - B^H(\lambda_f + YK^I - A^H \tilde{K}(X_f + YK^I) -
$$

$$
A^H \dot{\xi}(X_f + YK^I) - C_{JK} \tilde{K}(X_f + YK^I) -
$$

$$
\int_0^1 \Phi^J f dm dx = 0
$$

$$
B^H \dot{v}^I - A^H \dot{w}^I - A^H \tilde{K}(X_f + YK^I) - C_{JK} \tilde{K}^I = 0
$$

$$
A^H \dot{\xi}(X_f + YK^I) - C_{JK} \tilde{K}^I = 0
$$

$$
A^H \dot{\xi} + B^H \dot{w}^I - A^H \tilde{K}(X_f + YK^I) - C_{JK} \tilde{K}^I = 0
$$

where $\lambda = \frac{dx}{d\xi}$, $A^H = \int_0^1 N^I(\xi) N^I(\xi) d\xi$, $B^H = \int_0^1 N^I(\xi) \frac{d[N^I(\xi)]}{d\xi} d\xi$, $C_{\tilde{K}^I} = \int_0^1 N(\xi) N^I(\xi) N^I(\xi) d\xi$.

$\xi(x)$ is a natural coordinate in an element assuming that

$$
\rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = \rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = 0
$$

$$
\rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = \rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = 0
$$

$$
\rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = \rho \ddot{\bar{z}} + C_{\bar{z}} \dot{\bar{z}} + M_{\bar{z}} \bar{z} = 0
$$

where Eqs.(18)-(20), Eqs.(21)-(22), Eqs.(23)-(24) are for lift, moment and drag acting at the forward quarter chord, respectively. $W_0$ and $W_1$ denote the speed induced by the effective angle of attack at the forward quarter chord and the airfoil rotation, respectively. The various parameters are determined by parameter identification approach based on wind tunnel test data. For the generic application, the detailed information about various coefficients and parameters can be found in Refs.[19],[20].

In the present study, assume that, for a symmetrical airfoil, both the lift and moment loops are centrosymmetric about the coordinate origin and the drag loop is axisymmetric about the vertical axis when the variations of the effective angle of attack, pitch angle and oncoming flow velocity are identical in both of the positive and negative regions of the effective angle of attack.

2.4. Combining structural model with aerodynamic model

Here the structural model is combined with the aerodynamic model in an element where $I, J=1, 2$. As-
sume that the various strips have no influence aerodynamically upon each other, which is valid if the wing is slender and beamlike and has been successfully applied to aeroelastic investigations of high-aspect-ratio wings [1-2,4,8,11-13].

The first step is to express the variables \( W_0 \) and \( W_1 \) in ONERA (Edlin) model as follows:

\[
W_0 = V_1 \alpha, \quad W_1 = b \dot{\theta}_p
\]

(25)

where \( \alpha \) is the effective angle of attack at the aerodynamic center (the forward quarter chord), that is, \( \alpha = A_1 \arctan( - \frac{V_3}{V_2} ) \), here, \( A_1 = 180 / \pi \), \( \frac{V_3}{V_2} = \left[ V_1 - y_{ae} O_3 \right] \frac{V_2}{V_2 + y_{ae} O_1} \), \( y_{ae} \) is the offset from the elastic axis of the aerodynamic center, \( V_1 = \sqrt{\frac{V_2}{V_3} + \frac{V_3^2}{V_2}} \), \( b \) is the semi-chord, \( \dot{\theta}_p \) is the pitch angular velocity and \( \dot{\theta}_p = A_1 O_3 \).

Fig.1 illustrates the aerodynamic loads acting on the airfoil. Aerodynamic loads per unit length are

\[
f_{\text{aero}} = \begin{bmatrix}
0 & 0 & 0 \\
\sin \alpha & 0 & -\cos \alpha \\
\cos \alpha & 1 & \sin \alpha
\end{bmatrix}
\]

(26)

\[
m_{\text{aero}} = \begin{bmatrix}
M + y_{ae} a_{\text{aero}} \\
0 \\
0
\end{bmatrix}
\]

(27)

where \( L_C \) and \( L_{NC} \) are circulatory and non-circulatory parts of lift, which, according to Eq.(18), are expressed by

\[
L_C = \frac{1}{2} \rho S V_1 (\Gamma_{11} + \Gamma_{12})
\]

(28)

\[
L_{NC} = \frac{1}{2} \rho S (s b W_0 + k_b W_1)
\]

(29)

![Diagram of Aerodynamic loads acting on airfoil.](image)

According to Eq.(21), Eq.(23), Eqs.(25)-(29), one can have the expression of aerodynamic contributions:

\[
h J^{-1} \Phi^J \begin{bmatrix}
 f_{\text{aero}} \\
m_{\text{aero}}
\end{bmatrix} d\xi = a_{\text{aero}} M_{\text{aero}} f_{\text{aero}}^J + a_{\text{aero}} f_{\text{aero}}^J
\]

(30)

In addition, according to Eqs.(19)-(20), Eq.(22) and Eq.(24), one can have the following aerodynamic circulation equation:

\[
a_{\text{aero}} M_{\text{aero}}^J f_{\text{aero}}^J = a_{\text{aero}} f_{\text{aero}}^J
\]

(31)

which are directly applied to each node, and thus repeated indices are not summed. Here, \( q_{a_{\text{aero}}}^J f_{\text{aero}}^J = [v^J w^J \Gamma^J]^T \), \( \text{aero} M_{\text{aero}}^J \) is a \( 7 \times 13 \) matrix, \( \Gamma^J = [\Gamma_{11}^J \Gamma_{12}^J \Gamma_{13}^J \Gamma_{1m}^J \Gamma_{1m^2}^J \Gamma_{1m^3}^J \Gamma_{1m}^J \Gamma_{1m^2}^J \Gamma_{1m^3}^J]^T \), \( \Gamma_{1m}^J = \Gamma_{1m^2}^J \), \( \Gamma_{1m^3}^J = \Gamma_{1d^2}^J = 1 \).

2.5. Gravity loads

According to Refs.[21],[22], gravity loads per unit length are expressed as

\[
f^g = \bar{G} \mu g
\]

\[
m^g = \bar{G} \mu \bar{g} g
\]

(32)

where \( \bar{G} \) is the gravity acceleration constant, \( g \) the gravity vector expressed in the cross-sectional reference frame of the deformed beam. The gravity vectors have to satisfy the following differential equations in space and time:

\[
g + \dot{\theta} \bar{g} = 0
\]

\[
g + \dot{\theta} (\bar{k} + \bar{h}) \bar{g} = 0
\]

(33)

This article supposes a level flight at a constant speed in the symmetry plane of fuselage. Thus, the gravity vector at the root node is

\[
g^J = [0 - \sin \alpha_{\text{root}} - \cos \alpha_{\text{root}}]^T
\]

(34)

where \( \alpha_{\text{root}} \) is the effective angle of attack at the wing root. And according to Eq.(33) and the constraint of constant magnitude, by using the central difference method the other gravity vectors at other nodes are

\[
g^{J-1} = [A + h(\bar{k} + \bar{r}^J)]^{-1} [A - h(\bar{k} + \bar{r}^J)] g^J
\]

(35)

where \( J = 1,2, \ldots, n \); \( \bar{r}^J = \frac{(y^{J+1} - y^J)}{2} \).

Based on Eq.(32), Eqs.(34)-(35), one can have the expression of gravitational contributions:

\[
h J^{-1} \Phi^J \begin{bmatrix}
f^g \\
m^g
\end{bmatrix} d\xi = g J f_{\text{aero}}^J
\]

(36)

2.6. Nonlinear aeroelastic formulations

Based on Eq.(17), Eqs.(30)-(31) and Eq.(36), the elemental aeroelastic equations can be represented by

\[
\begin{bmatrix}
\text{stru aero} M_{\text{aero}}^J \quad \text{stru aero} M_{\text{aero}}^J \\
\text{stru aero} M_{\text{aero}}^J 
\end{bmatrix} q_{\text{aero}}^J f_{\text{aero}}^J = \begin{bmatrix}
\text{stru aero} f_{\text{aero}}^J + a_{\text{aero}} f_{\text{aero}}^J + g J f_{\text{aero}}^J \\
\text{stru aero} f_{\text{aero}}^J
\end{bmatrix}
\]

(37)

\[
\begin{bmatrix}
\text{stru aero} M_{\text{aero}}^J \\
\text{stru aero} M_{\text{aero}}^J \\
\text{stru aero} M_{\text{aero}}^J
\end{bmatrix} q_{\text{stru aero}}^J f_{\text{stru aero}}^J = \begin{bmatrix}
\text{stru aero} f_{\text{stru aero}}^J \\
\text{stru aero} f_{\text{stru aero}}^J \\
\text{stru aero} f_{\text{stru aero}}^J
\end{bmatrix}
\]

(38)

where \( \text{stru aero} M_{\text{aero}}^J = [\text{stru aero} M_{\text{aero}}^J - \text{stru aero} M_{\text{aero}}^J 0_{6 \times 7}] \), \( \text{aero} M_{\text{aero}}^J = \begin{bmatrix}
\text{aero} M_{\text{aero}}^J \\
J = 1 \\
J = 1 \ldots, n, J = 1, 2
\end{bmatrix}
\]

By assembling multiple elemental equations one can obtain the nonlinear aeroelastic formulations for high-aspect-ratio wings with 19 degrees of freedom at each node. The boundary conditions are
\[ V(0,t) = V^l, \quad \Omega(0,t) = 0 \]
\[ F(l,t) = 0, \quad M(l,t) = 0 \]  \hspace{1cm} (39)

where \( V^l = [0 \quad V_x \cos \alpha_{root} \quad -V_x \sin \alpha_{root}]^T \) and \( V_x \) is the level flight speed.

The position and geometry of the wing at arbitrary moment can be obtained based on generalized strain results\(^{[21]}\).

### 3. Numerical Results

Eqs.(37)-(38) indicate that the aeroelastic formulations have such form as

\[ Mq = f(q,t) \]  \hspace{1cm} (40)

which is suitable for the use of the MATLAB ordinary differential equation solver\(^{[23]}\). The nonlinearity terms that can only be expressed in integral form are calculated using Gaussian quadrature method. As \( M \) is a block tridiagonal matrix, the block chase method is used to solve the block tridiagonal linear systems.

In the present study, it is assumed that the wing is undeformed at the initial moment of time-domain simulation while levelly flying at a constant speed. Firstly, the Goland wing\(^{[24]}\) is analyzed to validate the present model. Then, examples of effects of nonlinearities, flight speed, and drag on aeroelastic behaviors of high-aspect-ratio flexible wings are presented.

#### 3.1. Goland wing

The flutter speed of Goland wing given by M. Goland is 393 mph (175.7 m/s) with the effects of gravity load neglected\(^{[24]}\). Fig.2 shows the tip response of Goland wing at various flight speeds calculated based on the present model without gravity loads considered. The amplitude of oscillation decays at flight speed of 160.0 m/s while grows at 180.0 m/s. A critical state, i.e. a periodic oscillation, is observed at flight speed of 170.6 m/s that should be the flutter speed predicted by the present model. The error relative to that given by Goland is 2.9%.

#### 3.2. High-aspect-ratio flexible wings

Table 1 lists the planform and structural data of a high-aspect-ratio flexible wing model\(^{[11]}\) with airfoil being NACA 0012.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half span/m</td>
<td>16</td>
</tr>
<tr>
<td>Chord/m</td>
<td>1</td>
</tr>
<tr>
<td>Spanwise elastic axis</td>
<td>50% of chord</td>
</tr>
<tr>
<td>Gravity center</td>
<td>50% of chord</td>
</tr>
<tr>
<td>Mass per unit length/(kg·m(^{-1}))</td>
<td>6.067</td>
</tr>
<tr>
<td>Mass moment of inertia/(kg·m)</td>
<td>0.207</td>
</tr>
<tr>
<td>Torsional rigidity/(N·m(^2))</td>
<td>3.240×10(^4)</td>
</tr>
<tr>
<td>Bending rigidity (flapwise)/(N·m(^2))</td>
<td>1.448×10(^5)</td>
</tr>
<tr>
<td>Bending rigidity (chordwise)/(N·m(^2))</td>
<td>1.275×10(^7)</td>
</tr>
</tbody>
</table>

The time histories of oscillations with gravity loads considered at flight speeds of 63.0 m/s and 64.0 m/s were obtained. The tip vertical and horizontal displacements, the tip effective angle of attack, and the tip twist are plotted against time in Fig.3. The wing oscillates with exponential growth in amplitude until the LCO is reached due to the increasingly nonlinear stiffness. However, the LCO is very different at the two different speeds. During the time history of oscillations at speed of 63.0 m/s, the tip effective angle of attack, of which the maximum value is greater than that at any other spanwise position, never exceeds 11°. And the stall angle of attack at speeds of 50-70 m/s predicted by ONERA (Edlin) dynamic stall model ranges 14.16°-14.57°, so the aerodynamic stall does not occur and the three aerodynamic circulation variables associated with the stall for lift, moment, and drag keep zero during the time history. This result indicates that the geometric stiffness arising mainly from structural geometrical nonlinearities could lead to the LCO without stall occurring. When the speed increases to 64.0 m/s, stall occurs and the characteristics of the LCO become complicated. The time history of this LCO shows clearly that near the moment when the effective angle of attack reaches the maximum and stall occurs, the vertical and horizontal deflections of the wing decreases to the minimum. After entering the
stall region, because of increasing drops of the aerodynamics (the effects of the gradual rise of drag could be omitted), the twist and vertical deflections gradually decrease, and thus the effective angle of attack is reduced gradually. Until getting clear of stall, the excitation effects of aerodynamics begin to increase with the growth in the amplitudes of oscillations and the maximum of the effective angle of attack. When stall occurs again, such oscillation process is repeated. Dynamic stall should dominate this LCO behavior.

To investigate the effects of flight speed on the LCO behavior, the time histories of oscillations without gravity loads considered at flight speeds of 58.0 m/s, 59.5 m/s and 60.0 m/s were obtained. The LCO behaviors at speeds of 58.0 m/s and 60.0 m/s are respectively similar to those at speeds of 63.0 m/s and 64.0 m/s with gravity loads considered. Figs.4-6 illustrate phase plane plots of wing tip motion after LCO having occurred at various speeds. As speed increases,
there are found increases in the amplitude and the complexity of the LCO due to the increasingly excitation effects of aerodynamics and effects of dynamic stall.

In order to clarify the effects of drag on the aeroelastic behavior, the time histories of oscillations without drag considered at flight speeds of 60.0 m/s and 61.0 m/s were obtained without including gravity loads. The results are compared with those with drag considered and shown in Figs. 7-8 with the plots of tip vertical displacement and twist. It is clear from Fig. 7 that the LCO amplitude with drag considered is much greater than that without it, which means the excitation effects of aerodynamics are stronger in the case of taking drags into account. This suggests that it could be necessary to include drag to exactly predict the onset speed of the flutter or LCO for high-aspect-ratio flexible wings; otherwise the predicted result would be lower. Fig. 8 indicates that the LCO without drag considered is quite similar to that with it yet having some differences between the two responses caused by the different excitation effects of aerodynamics. Consequently, whether the effects of drag are taken into account does not bring changes to LCO type. Moreover, the effects of dynamic stall of drag, that is, the rise of drag due to stall, on the LCO type should be negligible.
4. Conclusions

A first-order, state-space nonlinear aeroelastic model for high-aspect-ratio flexible wings has been developed by combining a geometrically exact, nonlinear anisotropic beam model with nonlinear ONERA (Edlin) dynamic stall model with drag considered. It takes into account material anisotropy, structural geometrical nonlinearities and dynamic stall. Numerical examples of effects of nonlinearities, flight speed, and drag on aeroelastic behaviors of high-aspect-ratio flexible wings have been presented.

As flight speed increases, the amplitude and the complexity of the LCO increase and dynamic stall becomes increasingly important. When flight speed is relatively low and stall does not occur, geometric stiffness arising mainly due to structural geometrical nonlinearities could lead to the LCO. When flight speed is relatively high and stall occurs, dynamic stall dominates the LCO behavior. In addition, the effects of drag on the LCO type could be neglected, but drag should be taken into account if it is required to exactly predict the onset speed of the flutter or LCO of high-aspect-ratio flexible wings.

Acknowledgements

The authors would like to acknowledge Prof. Mayuresh J. Patil and Dr. Chong-Seok Chang for clarifying Hodges’ fully intrinsic theory.

References


Biographies:

Zhang Jian Born in 1981, he received B.S. degree from Beijing University of Aeronautics and Astronautics in 2004 and then became a Ph.D. candidate at the same institute. His main research interests lie in aeroelasticity and flight dynamics.
E-mail: jaci_me@ase.buaa.edu.cn

Xiang Jinwu Born in 1964, he is a professor and doctoral supervisor in School of Aeronautic Science and Engineering, Beijing University of Aeronautics and Astronautics. His main research interests include aircraft design, aeroelasticity and structural dynamics, etc.
E-mail: xiangjwbj@sina.com.cn