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Physics Procedia 33 (2012) 1254 – 1260

Physics Procedia

2012 International Conference on Medical Physics and Biomedical Engineering

Controller Design of Quadrotor Aerial Robot

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Abstract

This paper deduced the nonlinear dynamic model of a quadrotor aerial robot, which was a VTOL (vertical tale-off and landing) unmanned air vehicle. Since that is a complex model with the highly nonlinear multivariable strongly coupled and under-actuated property, the controller design of it was very difficult. Aimed at attaining the excellent controller, the whole system can be divided into three interconnected parts: attitude subsystem, vertical subsystem, position subsystem. Then nonlinear control strategy of them has been described, such as SDRE and Backstepping. The controller design was presented to stabilize the whole system. Through simulation result indicates, the various models have shown that the control law stabilize a quadrotor aerial robot with good tracking performance and robotness of the system.

Keywords: quadrotor aerial robot; VTOL; SDRE; Backstepping

1. Introduce

Quadrotor aerial robot is a VTOL (vertical tale-off and landing) rotary UAV. Quadrotor aerial robot has adhesive attraction because compared with general single-rotor helicopter, its rotor is more small and highly improved the security of flight. It can avoid the danger of the exposed rotors pushing over the surrounding objects. Further more, the thrust which is generated by the four rotors of quadrotor aerial robot can easily realize static state hovering. One additional advantage of the quadrotor compared to a conventional helicopter is the simplified rotor mechanics. But the development of VTOL gyroplane UAV is slower [1-2]. The main reason is the control of VTOL aerial robot is very complex, control of autonomous flight is can not realize by the previous technology level. Recently, along with the

doi:10.1016/j.phpro.2012.05.207

advancement of new-style material, new-style energy source, flight control technology, micro-electronic technology, micro-inertial navigation technology and sensor technology, which is result in the rapidly developing of quadrotor aerial robot, and it gradually become a point of researchers attention

The primary mission of this paper is in the section 2 introduced the structure character of the quadrotor aerial robot. In the section 3 built the dynamics and kinematics system model. In the section 4, discussed the control algorithmic and designed the flight system controller. In the section 5 was presented the simulation result analysis. The last is conclusion.

2. Structure Character of Quadrotor Aerial Robot Airframe

Since 2007, Harbin engineering university independent innovation laboratory has researched the micro flight correlation technique airframe structure design and prototype making work. Four rotors section along with airframe and control board has composed this prototype. Among that, the rotors section including four DC brushless driven motor blades and connecting pieces. Airframe has been composed by two intersection aluminium alloy frames. Its weight is 900g. the maximum length in diameter is 64CM; maximum load is 200g; once charge the effective flight time: 6min. stable vertical take-off and landing, indoor fixed point horizontal drifting less than 1CM/s, vertical drifting less than 10CM/s; flight vibrating frequent less than 2° /s; the flight attitude including hovering ahead flight draw back flight left cross flight right cross flight rise descend. To avoid blade damaged or damage other object, design safety protection device including carbon fiber tube and plastic ropes.

DraganflyerIII blade and DC brushless motor has been adopted as power equipment. Airframe sensor including: MMA7260 which is a cheap singlechip triaxial acceleration transducer of American Freescale, ENC-03M which is a single-axial angle acceleration transducer of Japanese murata, HMC1052 which is a high performance double-magnetic resistance sensor one single chip electronic compass concentrate upon. Now the control system hardware section developed and tested has been finished. This system small volume light weight low system power consumption furthermore cheap and simple structure, be appropriate for researching all kinds of complex control algorithm. In the field of UAV autonomous navigation has wide application.The hardware structure of our quadrotor prototype is as shown in Fig. 1.

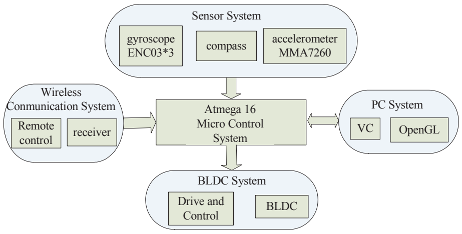

Fig.1structure of quadrotor aerial robot of independent innovation laboratory

3. Dynamics and Kinematics System Model

Defined two main reference frames as follows: The earth fixed inertial reference frame. O^e is a point which has been described as the moving vehicle at earth surface. X^e, Y^e are in the geography level and Z^e at vertical direction and points up which composes right handed coordinate system. The body fixed reference frame, which describes reference frame attached to the aerial robot. O^b is coincide on center of mass. Z^b is perpendicular to airframe surface and upward is the positive direction, initial position Z^b is common with force of gravity direction.

Looking O^e as original point, vector $\zeta = [x, y, z]^T$ and $\eta = [\phi, \theta, \psi]^T$ denote respectively translational positions and attitude angles of the quadrotor. The altitude angles $\{\phi, \theta, \psi\}$ are respectively called pitch angle $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$, roll angle $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ and yaw angle $(-\pi < \psi < \pi)$.

$V = [V_1, V_2, V_3]^T$, $\Omega = [\Omega_1, \Omega_2, \Omega_3]^T$ are respective translational velocities and rotational velocities of quadrotor aerial robot in body fixed reference frame. The velocities vector related between (V, Ω) and $(\dot{\zeta}, \dot{\eta})$ is follows:

$$\begin{cases} \dot{\zeta} = RV \\ \Omega = N\dot{\eta} \end{cases} \tag{1}$$

That two reference frames are not isolated. Earth frame can obtain through coordinate transformation of body frame. Orthogonal rotation matrix R_b^3 and N can be described as follows:

$$R_b^3 = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} N = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Using the kinematic equations and Newton's law, we can list following equations:

$$\dot{\zeta} = \upsilon_{\mathrm{e}} \tag{2}$$

$$\mathrm{m}\dot{\upsilon}_{\mathrm{e}} = \sum F_{\mathrm{tal}} \tag{3}$$

$$\dot{\eta} = \Omega_{\mathrm{e}} \tag{4}$$

$$I_f \dot{\Omega}_{\mathrm{e}} + \dot{\Omega}_{\mathrm{e}} \times I_f \dot{\Omega}_{\mathrm{e}} = \sum T_{tal} \tag{5}$$

Using dynamic equations, $\sum F_{\mathrm{tal}}$ and $\sum T_{\mathrm{tal}}$ can be calculated:

$$\sum F_{\mathrm{tal}} = -F_{\mathrm{e}} + F_a + F_g \tag{6}$$

$$\sum T_{\mathrm{tal}} = T - T_a - T_g \tag{7}$$

Where $I_f = \lfloor I_x,\ I_y,\ I_z \rfloor$ is the total inertia matrix of quadrotor aerial robot.

Using the above equations, the translation equations are given by:

$$\begin{cases} \ddot{\mathrm{x}} = u_1(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)/m - K_t\dot{x}/m \\ \ddot{\mathrm{y}} = u_1(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)/m - K_t\dot{y}/m \\ \ddot{\mathrm{z}} = u_1\cos\theta\cos\phi/m - K_t\dot{\mathrm{z}}/m - \mathrm{g} \end{cases} \tag{8}$$

And the rotation equations are given by:

$$\begin{cases} \ddot{\phi} = (I_z - I_y)\dot{\theta}\dot{\psi}/I_x - \mathrm{I_r}\dot{\theta}\mathrm{u}_5/I_x - K_r\dot{\phi} + d\mathrm{u}_2/I_x \\ \ddot{\theta} = (I_x - I_z)\dot{\phi}\dot{\psi}/I_y - \mathrm{I_r}\dot{\phi}\mathrm{u}_5/I_y - K_r\dot{\theta} + d\mathrm{u}_3/I_y \\ \ddot{\psi} = (I_y - \mathrm{I_x})\dot{\phi}\dot{\theta}/I_z - K_r\dot{\psi} + cu_4/I_z \end{cases} \tag{9}$$

4. Controller Design of Quadrotor Aerial Robot

4.1. Attitude Angle Control[3]

Cloutier has initially derived the state-dependent Riccati equation control [4]. The main idea was to solve problem by imitate linear quadratic regulation. Since SDRE has briefness design process, little calculate amount, better real-time, it has been wide applied to nonlinearity control field though it is a suboptimal method [5].

The SDRE method has been applied to the attitude control. For that problem $x_A^T = [x_1, x_2, x_3, x_4, x_5, x_6]^T \quad = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$ is the given state variables vector, while $u_A^T = [u_2, u_3, u_4, u_5]^T$ is the artificial input variables vector. Through factorizing (8), we can obtain the possible state-dependent model as follows:

Commanded：

$I_1 = I_z - I_y / I_x$ ， $I_2 = I_x - I_z / I_y$ ， $I_3 = I_y - I_x / I_z$,then:

$$\dot{x}_A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_6 I_1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & x_6 I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & x_2 I_3 & 0 & 0 \end{bmatrix} \cdot x_A + \begin{bmatrix} 0 & 0 & 0 & 0 \\ d/I_x & 0 & 0 & -I_r x_4 / I_x \\ 0 & 0 & 0 & 0 \\ 0 & d/I_y & 0 & -I_r x_2 / I_y \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c/I_z & 0 \end{bmatrix} \cdot u_A \qquad （10）$$

In (10) matrix $A(x_A)$ and $B(x_A)$ are all state-dependent matrix, then (8) can be written as

$$\dot{x}_A = A(x_A)x_A + B(x_A)u_A \qquad （11）$$

Using function (11) can calculate the control gain matrix $K(x_A)$, for the overall control input U1, it should be considered that a desired state $x_{A,d}$ which was given by the velocity control loop must be stabilized. If assuming that the control gain matrix $K(x_A)$ is already determined, a pre-filter matrix $M(x_A)$ can be guaranteed that a desired state which is not zero was stable:

$$M(x_A) = pinv((B(x_A)K(x_A) - A(x_A))^{-1} B(x_A)) \qquad （12）$$

Where pinv() denotes the pseudo-inverse of a non-quadratic matrix. From the above attitude control law, then the controller can be expressed as follows:

$$u_A = -K(x_A)x_A - M(x_A)x_{A,d} \qquad （13）$$

4.2. Backstepping Control of Z [6]

At first, defined state variable as:

$$\begin{cases} \mathrm{x}_1 = \mathrm{z} \\ \mathrm{x}_2 = \dot{x}_1 = \dot{\mathrm{z}} \end{cases} \qquad （14）$$

Dynamical model of the channel Z can be denoted is:

$$\begin{cases} \dot{\mathrm{x}}_1 = \mathrm{x}_2 = \dot{z} \\ \dot{\mathrm{x}}_2 = \ddot{z} = -g + u_1 \cos\phi \cos\theta / m \end{cases} \qquad （15）$$

Design step of backstepping controller are:

First step:

Defined a tracking error $\mathrm{z}_1 = \mathrm{x}_{1\mathrm{d}} - \mathrm{x}_1$， then its time derivative is：

$$\dot{\mathrm{z}}_1 = \dot{\mathrm{x}}_{1\mathrm{d}} - \dot{\mathrm{x}}_1 = \dot{\mathrm{x}}_{1\mathrm{d}} - \mathrm{x}_2 = \mathrm{z}_2 - \alpha_1 + \mathrm{f}_1 \qquad （16）$$

where， $z_2 = \alpha_1 - x_2$ is a tracking error, α_1 is an undetermined virtual input.

Choose candidated Lyapunov function is: $V_1 = \frac{1}{2} z_1^T z_1$

Its time derivative is:

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(z_2 - \alpha_1 + f_1) \qquad (17)$$

Introducing $\alpha_1 = c_1 z_1 + f_1$， where $c_1 > 0$ is a regulable parameter, then:

$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2$， considering stable of the closed-loop system, coupling item $z_1 z_2$ has been neglected.

$$\dot{z}_1 = -c_1 z_1 + z_2 \qquad (18)$$

The other variables respectively are:

$$\begin{aligned} f_1 &= \dot{z}_1 + x_2 = \dot{x}_{1d} \\ \alpha_1 &= c_1 z_1 + f_1 = c_1(x_{1d} - x_1) + \dot{x}_{1d} \\ z_2 &= \alpha_1 - x_2 = c_1(x_{1d} - x_1) + \dot{x}_{1d} - x_2 \end{aligned} \qquad (19)$$

Second step:

The time derivative of z_2 is:

$$\dot{z}_2 = c_1(\dot{x}_{1d} - \dot{x}_1) + \ddot{x}_{1d} - \dot{x}_2 = c_1(z_2 - c_1 z_1) + \ddot{x}_{1d} + g - u_1 \cos\phi \cos\theta / m$$

substituting $z_1 \sim z_2$ to the expression：$\dot{z}_2 = -z_1 - c_2 z_2$，control variable u_1 can be obtained by deducing.

$$\dot{z}_2 = c_1(z_2 - c_1 z_1) + \ddot{x}_{1d} + g - \frac{u_1}{m} \cos\phi \cos\theta = -z_1 - c_2 z_2$$

The Backstepping controller of channel Z is:

$$u_1 = \left[z_1 + g + c_1(z_2 - c_1 z_1) + c_2 z_2 + \ddot{z}_d \right] m / \cos\theta \cos\phi \qquad (20)$$

where， $c_1 > 0$， $c_2 > 0$ is constant

$$z_1 = x_1 - x_{1d} = z - z_d$$

$$z_2 = x_2 + c_1 z_1 - \dot{x}_{1d} = \dot{z} + c_1(z - z_d) - \dot{z}_d = c_1(z - z_d) - (\dot{z} - \dot{z}_d)$$

4.3. Position Control[7]

Position subsystem is given by the first two functions of (16). The friction item was be neglected, let $\dot{x}_d$ and $\dot{y}_d$ be respectively the desired speeds in x and y direction. Then error in desired and actual velocities is given as:

$$e_x = \dot{x}_d - \dot{x} \qquad (21)$$

$$e_x = \dot{x}_d - \dot{y} \qquad (22)$$

Thinking artificial input variable was composed of three independent first-order systems, then the control task was very easy, so lets look that as a pure proportion controller:

$$\ddot{x} = \tilde{u}_1 = K_x(\dot{x}_d - \dot{x}) = K_x e_x \qquad (23) \qquad \ddot{y} = \tilde{u}_2 = K_y(\dot{y}_d - \dot{y}) = K_y e_y \qquad (24)$$

According（23）and（24）, solving θ_d and ϕ_d of the position subsystem which has neglected the friction, then

$$\theta_d = \arcsin\left(U_x/\cos\psi\cos\phi - \sin\psi\sin\phi/\cos\psi\cos\phi\right) \quad (25)$$

$$\phi_d = arc\sin\left(\sin\psi U_x - \cos\psi U_y\right) \quad (26)$$

Where, U_x and U_y are:

$U_x = K_x e_x m/u_1 \qquad U_y = K_y e_y m/u_1$

Where K_x and K_y are positive constants and u_1 is desired vertical force input from altitude control.

5. Simulation Result

Aimed at verifying effectiveness and application effect of the control method, the simulation experiment has been carried on the quadrotor aerial robot. The parameters are respective as following: $I_x = I_y = I_z/2 = 1.2416\ Nm.s_2/rad$. m=0.9kg, d=0.64m, c=0.01m, g=9.81m/s^2 , $K_t = diag\left[10^{-2}, 10^{-2}, 10^{-2}\right] Ns/m$, $K_r = diag\left[10^{-3}, 10^{-3}, 10^{-3}\right] Nm.s/rad$, the control parameters as: c_1, c_2 is 2, and $K_x = 1.82, K_y = 3.71$.

The actual output and desired tracking of quadrotor aerial robot has been indicated in Fig.2. From which the actual output can track the desired tracking. Furthermore, from fig.3, we can see the optimal angle of inclination（ϕ and θ） motion and the minimal tracking error. From fig.4, indicates that the input signal is the realizability.

6. Conclusion

This paper analyses the dynamic and kinematics of quadrotor aerial robot point of view. Aimed at attaining the excellent controller, the whole system has been divided to three interconnected part: attitude subsystem, vertical subsystem, position subsystem. Then nonlinear control strategy for them has been described such as SDRE and Backstepping. Through simulation result indicated, the various models has shown that the control law stabilized a quadrotor aerial robot with good tracking performance and robotness of the system.

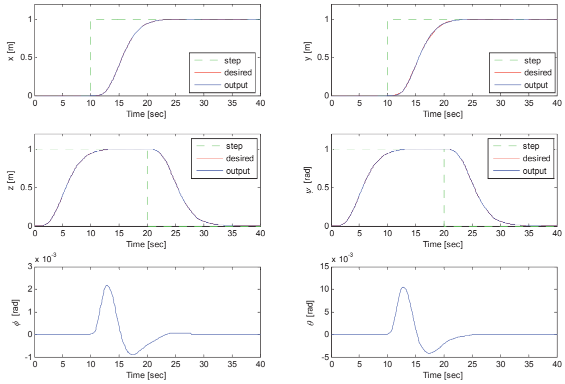

Fig.2 position of quadrotor aerial robot

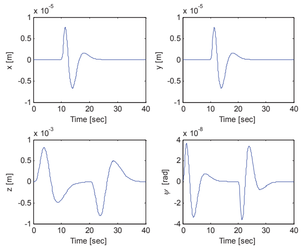

Fig.3 quadrotor tracking error

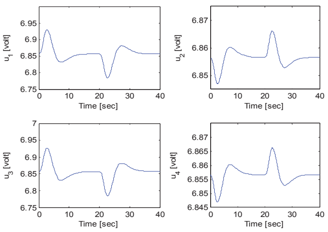

Fig.4 quadrotor input control

Acknowledgment

This paper is supported by the fundamental research funds for the central universities. The item number is: HEUCF101706.

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