Analysis of tangent hyperbolic nanofluid impinging on a stretching cylinder near the stagnation point

T. Salahuddin a,⇑, M.Y. Malik b, Arif Hussain b, Muhammad Awais b, Imad Khan b, Mair Khan b

⇑Mirpur University of Science and Technology (MUST), Mirpur 10250 (AJK), Pakistan
bDepartment of Mathematics, Quaid-i-Azam University, Islamabad 44000, Pakistan

A R T I C L E   I N F O

Article history:
Received 14 October 2016
Received in revised form 26 November 2016
Accepted 23 December 2016
Available online 05 January 2017

Keywords:
Stagnation point flow
Tangent hyperbolic nanofluid
Stretching cylinder
Heat generation/absorption
Boundary layer
Shooting method

A B S T R A C T

An analysis is executed to study the influence of heat generation/absorption on tangent hyperbolic nanofluid near the stagnation point over a stretching cylinder. In this study the developed model of a tangent hyperbolic nanofluid in boundary layer flow with Brownian motion and thermophoresis effects are discussed. The governing partial differential equations in terms of continuity, momentum, temperature and concentration are rehabilitated into ordinary differential form and then solved numerically using shooting method. The results specify that the addition of nanoparticles into the tangent hyperbolic fluid yields an increment in the skin friction coefficient and the heat transfer rate at the surface. Comparison of the present results with previously published literature is specified and found in good agreement. It is noticed that velocity profile reduces by enhancing Weissenberg number $\lambda$ and power law index $n$. The skin friction coefficient, local Nusselt number and local Sherwood number enhances for large values of stretching ratio parameter $k$.

Introduction

The layer in which the effects of viscosity are significant in the close neighborhood of the surface is titled as boundary layer. Ludwig Prandtl presented the concept of boundary layer on August 12, 1904 at the third international conference in Germany. He divided the fluid equations into two categories, one inside the boundary layer where the influence of viscosity is maximum and second outside the boundary area where the influence of boundary can be ignored. Boundary layers are pervasive in a great number of natural flows and fluid dynamics engineering applications, such as solicitations in liquid films in condensation procedure, polymer, extrusion of plastic sheets, glass blowing, spinning of fibers, cooling of elastic sheets, the aerodynamic extrusion of plastic sheets, paper manufacturing, the earth’s atmosphere, the surface of car, ship and air vehicles, etc. Sakiadis [1] was the forerunner who elaborated the concept of two dimensional boundary layer of Newtonian fluid towards a continuous moving solid surface. Crane [2] extended this concept into two-dimensional steady boundary layer of Newtonian fluid near the stagnation point over a stretching cylinder. In this study the developed model of a tangent hyperbolic nanofluid in boundary layer flow with Brownian motion and thermophoresis effects are discussed. The governing partial differential equations in terms of continuity, momentum, temperature and concentration are rehabilitated into ordinary differential form and then solved numerically using shooting method. The results specify that the addition of nanoparticles into the tangent hyperbolic fluid yields an increment in the skin friction coefficient and the heat transfer rate at the surface. Comparison of the present results with previously published literature is specified and found in good agreement. It is noticed that velocity profile reduces by enhancing Weissenberg number $\lambda$ and power law index $n$. The skin friction coefficient, local Nusselt number and local Sherwood number enhances for large values of stretching ratio parameter $k$.

parameters and plotted graphs for local Nusselt number and skin friction coefficient. The effect of variable heat flux on two dimensional steady boundary layer flow was deliberated by Lin et al. [4]. They generate the solution in terms of hypergeometric functions and concluded that for smaller Prandtl number the boundary layer is larger. Ali [5] investigated the boundary layer flow of a continuously stretched surface with buoyancy effects. The boundary layer stagnation point flow towards a stretching sheet was discussed by Nadeem et al. [6]. Again, Nadeem et al. [7] investigated the boundary layer flow of second grade fluid towards a stretching sheet with temperature dependent viscosity. Rangi et al. [8] studied the heat transfer and boundary layer flow towards a stretching cylinder with variable thermal conductivity.

The boundary layer flow of pseudoplastic fluids has great importance due to its extensive uses in solutions and melts of high molecular weight polymers, emulsion coated sheets like photographic films, polymer sheets, etc. Nadeem et al. [9] examined the influence of magnetic field and temperature dependent viscosity on the peristaltic flow of Newtonian incompressible fluid. Again, Nadeem et al. [10] studied the peristaltic flow of a magneto-hydrodynamic tangent hyperbolic fluid in a vertical asymmetric channel under long wavelength and low Reynolds number approximation. Akbar et al. [11] analyzed the effects of chemical reactions and heat transfer on tangent hyperbolic fluid treated through a tapered artery. Nadeem et al. [12] examined the peristaltic motion...

A base fluid comprises of nanoparticles deferred in conventional heat transfer basic fluid with the length scales of 1–100 nm is called nanofluid. The nanofluid enhances the convective heat transfer coefficient and thermal conductivity of the base fluids. Oil, water and ethylene glycol are poor conductors of heat. In order to increase the thermal conductivity of these fluids many techniques have been taken in description. One of these techniques is the addition of nano-sized material particles in the liquid. Choi et al. [14] found that with the addition of the nanoparticles the thermal conductivity of the base fluid enhances twice. Das [15] concluded that with the inclusion of nanoparticles the thermal conductivity becomes temperature dependent. Buongiorno et al. [16] evaluated the nanofluid coolants for advanced nuclear power plants. Malik et al. [17] analyzed the boundary-layer mixed convective flow of a nanofluid over a stretching plate. Khan et al. [18] investigated the numerical solution of nanofluid over a flat stretching surface. Akbar et al. [19] studied the MHD flow of nanofluid over a vertical exponentially stretching cylinder located at \( r = R \), where \( r \) is the coordinate normal to cylinder. The cylinder is stretched with two equivalent and conflicting forces along \( x \)-axis with the velocity \( u = \frac{bx}{T} \), by keeping the origin fixed as shown in Fig. 1. It is assumed that concentration and temperature at the wall is maintained at constant concentration \( C_w \) and temperature \( T_w \). Where \( C_{∞} \) and \( T_{∞} \) are ambient concentration and temperature respectively. Under the boundary layer approximation, the governing equations of the mass, momentum, energy and nanoparticles in cylindrical coordinates \( x \) and \( r \) are written as Ref. [12]

\[
\begin{align*}
\frac{\partial (r u)}{\partial x} + \frac{\partial (r v)}{\partial r} &= 0, \\
n \frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial u}{\partial r} &= \nu \left(1 - n\right) \frac{\partial^2 u}{\partial r^2} + \left(1 - n\right) \frac{n U_e}{r} + \frac{n}{\sqrt{2\nu}} \frac{\partial \partial u}{\partial r} + \frac{n}{\sqrt{2\nu}} \frac{\partial^2 u}{\partial r^2} \\
&+ \mu \frac{\partial u}{\partial x}, \\
\rho c_p \left( \frac{\partial \nu}{\partial r} + \frac{v}{r} \frac{\partial \nu}{\partial r} \right) &= \frac{\partial^2 C}{\partial r^2} + D_v \frac{\partial C}{\partial x} + \frac{D_v}{T_{∞}} \frac{\partial^2 C}{\partial r^2}, \\
\frac{\partial C}{\partial x} + \frac{v}{r} \frac{\partial C}{\partial r} &= \frac{D_v}{r} \frac{\partial C}{\partial x} + \frac{D_v}{T_{∞}} \frac{\partial^2 C}{\partial r^2}.
\end{align*}
\]

The associated boundary conditions are

\[
\begin{align*}
u &= \frac{ax}{T}, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad r = R, \\
u &= \frac{U_v(x)}{T}, \quad v = 0, \quad T = T_{∞}, \quad C = C_{∞}, \quad \text{as} \quad r \to \infty.
\end{align*}
\]

Here \( u \) and \( v \) are the velocity components along \( x \) and \( r \) directions respectively, \( Q \) is the temperature dependent volumetric rate of heat source when \( Q > 0 \) and heat sink when \( Q < 0 \), dealing with the situation of exothermic and endothermic chemical reactions respectively, \( \nu \) is the kinematic viscosity, \( \rho \) is the density, \( A \) is the

**Mathematical formulation**

Consider steady two-dimensional stagnation point flow of optically dense tangent hyperbolic nanofluid towards a stretching cylinder with variable thermal conductivity and heat transfer characteristics with combined effects of temperature dependent internal heat generation/absorption and thermal buoyancy. Noghrehbadi et al. [21] examined the boundary layer flow of nanoparticles volume fraction with slip effects. Akbar et al. [22] analyzed the magnetic field analysis past over a stretching sheet in a suspension of gyrotactic microorganisms and nanoparticles. Akbar et al. [23] investigated the Double-diffusive natural convective MHD boundary-layer flow of a nanofluid over a stretching sheet.

Heat generation/absorption is reflected as a significant factor in numerous physical glitches such as fluids having endothermic and exothermic chemical reactions. Heat generation/absorption is assumed to be temperature-dependent or space dependent. Lawrence et al. [24] investigated the boundary layer flow of viscoelastic fluid over a stretching sheet with internal heat generation/exothermic reactions. Pavithra et al. [25] analyzed the heat transfer boundary layer flow of a dusty fluid over an exponentially stretching surface with combined effects of internal heat generation/absorption and viscous dissipation. Noghrehbadi et al. [26] studied the entropy generation of a nanofluid over an isothermal linear stretching plate with heat generation/absorption. Dessie et al. [27] examined the MHD boundary layer flow of Newtonian fluid with variable viscosity through a porous medium over a stretching sheet with viscous dissipation, heat source/sink and heat generation/absorption. Hakeem et al. [28] studied the effect of partial slip on hydromagnetic boundary layer flow in porous medium over a stretching surface with temperature and space dependent internal heat generation/exothermic reactions. Akbar et al. [29] examined the effects of thermal radiation and variable thermal conductivity on the flow of CNTs over a stretching sheet with convective slip boundary conditions. Akbar et al. [30] examined the two dimensional Magnetohydrodynamics flow of Eyring-Powell fluid. They noticed that by enhancing the intensity of the magnetic field as well as Eyring-Powell fluid parameter \( \gamma \) decrease the velocity profile. Mehmoond et al. [31] depicted the Non Aligned point flow and heat transfer of an Ethylene–Glycol and nanofluid towards stretching sheet. They noticed that Ethylene-based nanofluids have higher local heat flux than water-based nanofluids. Rana et al. [32] investigated the mixed convective oblique flow of a casson fluid with partial slip, internal heating and homogeneous–heterogeneous reactions.

The aim of the present analysis is to examine numerically, the stagnation point flow of tangent hyperbolic nanofluid over a stretching cylinder. Buongiorno model is used to examine the heat transfer due to nanoparticles. Moreover, heat generation/absorption effects are encountered for tangent hyperbolic nanofluid. For this persistence proper similarity transformations are used to diminish governing equations to ordinary differential equations.

The effect of thermophoresis, Brownian motion and Lewis number for nanofluid are deliberated through graphs. The near wall quantities, such as Sherwood number due to nanoparticle concentration, Nusselt number due to heat transfer and local skin friction for velocity profile are discussed.
stretching ratio parameter, $\Gamma$ is the Williamson parameter, $n$ the power law index, $D_T$ is the Brownian diffusion coefficient, $c_p$ is the specific heat, $\tau = \frac{\langle \gamma \rangle}{\langle \gamma \rangle_c}$ is the ratio between the effective heat capacity of the nanoparticles material and heat capacity of the fluid, $D_c$ is the thermophoresis diffusion coefficient and $C$ is resealed nanoparticles volume fraction.

Using the similarity transformations

$$\begin{align*}
\eta &= \sqrt{\frac{f''R^2}{M}}, \\
\psi &= \sqrt{\frac{aR}{M}} f(\eta), \\
\theta(\eta) &= \frac{T-T_\infty}{T_w-T_\infty}, \\
h(\eta) &= \frac{C_e-C_w}{C_w-C_e}, \\
u &= \frac{1}{R}, \\
v &= -\frac{1}{R}. 
\end{align*}$$

Eqs. (1)–(5) takes the form

$$(1-n)(1+2K\eta)f'' + ff' - (f')^2 + 2K(1-n)f'' + 2m(1+2K\eta)f'' + 3K(1+2K\eta)(f')^2 + A^2 = 0,$$  

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad h(0) = 1.$$  

where $K, \lambda, \beta, N_b, N_t, Le, A$ and Pr denotes curvature parameter, dimensionless Weissenberg number, heat generation/absorption, Brownian motion parameter, thermophoresis parameter, Lewis number, stretching ratio parameter and Prandtl number which are given by

$$Pr = \frac{a}{T}, \quad \lambda = \frac{a}{T}, \quad \beta = \frac{a}{T}, \quad A = \frac{a}{T},$$

$$N_b = \frac{\partial h}{\partial T} \left( \frac{C_e-C_w}{C_w-C_e} \right), \quad N_t = \frac{\partial h}{\partial T} \left( \frac{C_e-C_w}{C_w-C_e} \right),$$

$$Le = \frac{a}{T}, \quad \beta = \frac{a}{T}.$$  

Skin friction coefficient $C_f$, local Nusselt number $Nu_a$ and the local Sherwood number $Sh_a$ are defined as:

$$C_f = \frac{\tau_w}{\mu_0}, \quad Nu_a = \frac{xq_w}{k(T_f-T_\infty)}, \quad Sh_a = \frac{xh_m}{D_h(C_w-C_e)},$$

$$\tau_w = \mu \left( \frac{1}{R} \frac{\partial u}{\partial r} + \frac{n \Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial r} \right)^2 \right)_{r=R},$$

$$q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r=R}, \quad h_m = -k \left( \frac{\partial T}{\partial r} \right)_{r=R}.$$  

while the dimensionless forms of skin friction coefficient, local Nusselt number and Sherwood number are

$$\frac{C_f \mathrm{Re}_n^{1/2}}{2} = (1-n) f''(0) + n \sqrt{2} f'(0), \quad \frac{Nu_a}{\mathrm{Re}_n^{1/2}} = -\theta'(0),$$

$$\frac{Sh_a}{\mathrm{Re}_n^{1/2}} = -h'(0).$$

where $\mathrm{Re}_n = a^{1/2} n^{1/2} \lambda^{1/2}.$

**Numerical solutions**

The nonlinear coupled ordinary differential equations (7)–(9) are solved by using shooting method. The step size $\Delta \eta = 0.1$ is chosen to obtain numerical solution. As the energy, momentum and concentration equations are of order second, third and second respectively. After converting Eqs. (7)–(9) into first-order equations takes the form

$$y_1 = y_2, \quad y_3 = y_5,$$  

$$y_4' = \left( \frac{y_2 y_3 - y_1 y_4 - 3K\lambda(1+2K\eta)^{1/2}}{(1-n)(1+2K\eta)^{1/2}} \right),$$

$$y_5' = \left( -2K\lambda y_2 - Pr y_4 - N_t(1+2K\eta)N_t y_5 + N_t(1+2K\eta)(y_3)^2 - Pr y_4(y_4) \right) \left( 1+2K\eta \right).$$

The boundary conditions become

$$y_1 = 0, \quad y_2 = 1, \quad y_4 = 1, \quad y_5 = 1, \quad \text{at} \ \eta = 0,$$

$$y_2 - A, \quad y_4 - 0, \quad y_5 - 0, \quad \text{as} \ \eta \to \infty.$$  

Here four conditions are known and three of them are unknown. Donating the unknown initial conditions by $U_1, U_2$ and $U_3$,

$$y_1 = 0, \quad y_2 = 1, \quad y_3 = U_1, \quad y_4 = 1, \quad y_5 = U_2, \quad 6 = 1,$$

$$y_3 = U_3 \ \text{at} \ \eta = 0.$$  

Eqs. (15)–(21) are solved with the help of initial conditions defined in Eq. (23). The computed boundary values at $\eta = 4$ depends on the choice of $U_1, U_2$ and $U_3$. The accurate choice of $U_1, U_2$ and $U_3$ delivers the given boundary conditions at $\eta = 4$ that is it satisfies the Eq. (24)(boundary residual).

$$\Phi_1(U_1, U_2, U_3) = 0.1, \quad \Phi_2(U_1, U_2, U_3) = 0. \quad \Phi_3(U_1, U_2, U_3) = 0.$$

If $U_1, U_2$ and $U_3$ does not satisfy the boundary residual then their values will be refined by using Newton–Raphson method.

**Discussion**

Shooting method is used to solve the nonlinear ordinary differential equations (7)–(9) with boundary conditions Eq. (10) and the behavior of curvature parameter $K$, stretching ratio parameter $\Lambda$, Brownian motion parameter $N_b$, thermophoresis parameter $N_t$, heat generation/absorption $\beta$, Prandtl number $Pr$, power law index $n$, Weissenberg number $\lambda$ and Lewis number $Le$ on velocity, temperature and concentration profiles are illustrated through graphs. In order to check the accuracy of the solution, the values of $f''(0)$ are calculated for unlike values of stretching ratio parameter $\Lambda$, by ignoring the effects of Lewis number, curvature parameter, thermophoresis parameter, dimensionless Weissenberg number, Brownian motion parameter and heat generation/absorption. The results are compared with the values calculated by Mahapatra.
Table 1
Comparison of $f''(0)$ with the previous existing literature when $n=0$, $\lambda = 0$, $\beta = 0$, $N_t = 0$, $N_b = 0$, $L_e = 0$, $\alpha = 0$, $\delta = 0$ and $K = 0$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>Mahapatra [27]</th>
<th>Ibrahim [28]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.9980</td>
<td>-0.9980</td>
<td>-0.9980</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.5694</td>
<td>-0.5694</td>
<td>-0.5694</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.9181</td>
<td>-0.9181</td>
<td>-0.9181</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.6673</td>
<td>-0.6673</td>
<td>-0.6673</td>
</tr>
<tr>
<td>$2$</td>
<td>2.0175</td>
<td>2.0175</td>
<td>2.0175</td>
</tr>
<tr>
<td>$3$</td>
<td>4.7292</td>
<td>4.7292</td>
<td>4.7292</td>
</tr>
</tbody>
</table>

Table 2
Comparison of results local Nusselt number $-\frac{\theta'(0)}{C_0}$ for several values of $Pr$ and $A$ when $n=0$, $\lambda = 0$, $N_t = 0$, $N_b = 0$, $L_e = 0$, $\alpha = 0$, $\delta = 0$ and $K = 0$.

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$A$</th>
<th>Mahapatra [27]</th>
<th>Ibrahim [28]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$0.1$</td>
<td>0.603</td>
<td>0.6022</td>
<td>0.6022</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.625</td>
<td>0.6246</td>
<td>0.6247</td>
<td></td>
</tr>
<tr>
<td>$0.5$</td>
<td>0.692</td>
<td>0.6924</td>
<td>0.6927</td>
<td></td>
</tr>
<tr>
<td>$1.5$</td>
<td>$0.1$</td>
<td>0.7777</td>
<td>0.7768</td>
<td>0.7776</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.797</td>
<td>0.7971</td>
<td>0.7975</td>
<td></td>
</tr>
<tr>
<td>$0.5$</td>
<td>0.8648</td>
<td>0.8648</td>
<td>0.8648</td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Values of skin friction coefficient $\frac{C_f}{Re^{1/2}}$ with respect to $A$, $\lambda$, $n$ and $K$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$K$</th>
<th>$\lambda$</th>
<th>$n$</th>
<th>$(1-n)f''(0) + n\eta f'^2(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1$</td>
<td>$0.1$</td>
<td>$0.1$</td>
<td>$0.1$</td>
<td>$-0.5546$</td>
</tr>
<tr>
<td>$0.2$</td>
<td>$-0.9020$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td>$-0.8333$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$-0.5546$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.2$</td>
<td>$-0.8984$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td>$-1.0243$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$-0.5546$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.2$</td>
<td>$-0.5540$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td>$-0.5535$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$-0.5546$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.2$</td>
<td>$-0.8967$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td>$-0.8355$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4
Values of local Nusselt number $\frac{Nu}{L_e}$ with respect to $N_b$, $N_t$, $\beta$ and $Pr$.

<table>
<thead>
<tr>
<th>$N_b$</th>
<th>$N_t$</th>
<th>$\beta$</th>
<th>$Pr$</th>
<th>$-\frac{\theta'(0)}{C_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1$</td>
<td>$0.1$</td>
<td>$1.2$</td>
<td>$0.1$</td>
<td>0.5565</td>
</tr>
<tr>
<td>$0.2$</td>
<td></td>
<td></td>
<td>$0.5205$</td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td></td>
<td></td>
<td>$0.4858$</td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$0.1$</td>
<td>$1.2$</td>
<td>$0.5565$</td>
<td></td>
</tr>
<tr>
<td>$0.2$</td>
<td></td>
<td></td>
<td>$0.5296$</td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td></td>
<td></td>
<td>$0.5034$</td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$1.2$</td>
<td>$0.5565$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1.3$</td>
<td>$0.5745$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1.4$</td>
<td>$0.5921$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td></td>
<td>$0.5565$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.2$</td>
<td></td>
<td>$0.4467$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td></td>
<td>$0.3172$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5
Values of local Sherwood number $\frac{Sh}{L_e}$ with respect to $N_b$, $N_t$, $L_e$ and $Pr$.

<table>
<thead>
<tr>
<th>$N_b$</th>
<th>$L_e$</th>
<th>$N_t$</th>
<th>$Pr$</th>
<th>$-\frac{h''(0)}{K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1$</td>
<td>$1.2$</td>
<td>$0.1$</td>
<td>$1.2$</td>
<td>0.2514</td>
</tr>
<tr>
<td>$0.2$</td>
<td></td>
<td></td>
<td></td>
<td>0.3943</td>
</tr>
<tr>
<td>$0.3$</td>
<td></td>
<td></td>
<td></td>
<td>0.4415</td>
</tr>
<tr>
<td>$0.1$</td>
<td>$1.2$</td>
<td></td>
<td></td>
<td>0.2514</td>
</tr>
<tr>
<td>$1.3$</td>
<td></td>
<td></td>
<td></td>
<td>0.2980</td>
</tr>
<tr>
<td>$1.4$</td>
<td></td>
<td></td>
<td></td>
<td>0.3436</td>
</tr>
</tbody>
</table>

Tables 3–5 shows the behavior of the skin friction coefficient, local Nusselt number and local Sherwood number respectively for different values of parameters. It is observed that local Nusselt number is decreasing function while local Sherwood number is an increasing function for unlike values of dimensionless parameters $N_b$, $N_t$, $L_e$ and $Pr$. Fig. 2 presents the behavior of curvature parameter $K$ on velocity profile. It is evident from figure that velocity increases with increasing values of curvature parameter $K$. Near the surface of cylinder velocity reduces due to friction of the wall to the fluid.
particles. The augmentation in velocity is due to the fact that after increasing curvature parameter $K$, radius of curvature decreases which causes area of the cylinder to reduce. Hence less resistance is offered by the cylinder to the fluid, so velocity increases. Fig. 3 shows how stretching ratio parameter $A$ affects the velocity profile. It is noticed that when free stream velocity exceeds the stretching velocity of the cylinder, the fluid velocity increases and boundary layer thickness reduces with increase in stretching ratio parameter $A$. Moreover, when free stream velocity is less than the stretching velocity of the cylinder, the fluid velocity decreases and boundary layer thickness increases monotonically. Fig. 4 depicts the influence of Weissenberg number $\lambda$ on velocity profile. It is observed
from figure, reduction in velocity is noticed for each increment in value of Weissenberg number $\lambda$, because after increasing Weissenberg number $\lambda$, the relaxation time increases which offers more resistance to flow and hence velocity reduces. Fig. 5 shows the influence of power law index $n$ on velocity profile. It is quite interesting to observe that for large values of power law index $n$, the boundary layer thickness reduces. The effects of Brownian motion parameter $N_b$ on temperature profile is presented in Fig. 6. It is noticed from figure that, as the values of Brownian motion parameter $N_b$ increases, the random motion of fluid particles accelerates the collision between them and hence the temperature of the fluid increases. Moreover, the boundary layer thickness increases by
increasing the Brownian motion parameter $N_b$. The behavior of thermophoresis parameter $N_t$ on temperature profile may be examined from Fig. 7. As the values of thermophoresis parameter $N_t$ increases, the fluid particles move from hot region to cold region and hence temperature of the fluid increases. Moreover, the boundary layer thickness increases by increasing the thermophoresis parameter $N_t$. Fig. 8 demonstrates the variation in Brownian motion parameter $N_b$ on concentration profile. It is observed that by increasing the value of Brownian motion parameter $N_b$, the concentration boundary layer thickness reduces. Fig. 9 depicts the variation in thermophoresis parameter $N_t$ on concentration profile. It is observed that by increasing the values of thermophoresis parameter $N_t$, the concentration boundary layer thickness decreases. Fig. 10 shows the behavior of Lewis number...
Le on concentration profile. It is observed that by increasing the Lewis number Le the concentration profile reduces. Because by increasing the Lewis number Le the mass diffusivity reduces, so concentration profile decreases. Fig. 11 shows the behavior of heat generation/absorption $\beta$ on temperature profile. It is observed that by increasing heat generation/absorption $\beta$ kinetic energy of the fluid particles increases, so temperature and boundary layer thickness increases. Fig. 12 is plotted to see the effect of Prandtl number $Pr$ on temperature field. It is noticed that temperature field decreases with an increase in Prandtl number $Pr$. Because by increasing Prandtl number $Pr$, the thermal diffusivity of the fluid decreases, which causes decrease in temperature and boundary layer thickness. Fig. 13 shows the behavior of curvature parameter $A$ and velocity ratio parameter $A$ on skin friction coefficient. It is observed that the skin friction coefficient increases, as both parameters increases. Fig. 14 depicts the influence of Prandtl number $Pr$ and velocity ratio parameter $A$ on local Nusselt number. As the velocity ratio parameter $A$ and Prandtl number $Pr$ increases the heat transfer rate on the surface of cylinder increases. Fig. 15 shows the influence of Lewis number $Le$ and velocity ratio parameter $A$ on local Sherwood number. It is observed from figure that local Sherwood number increases as both parameters increases. Figs. 16–18 are plotted to see the flow pattern of fluid particles for different values of stretching ratio parameter $A$. It can be seen from these figures that by increasing stretching ratio parameter $A$ the stagnation region (the region where fluid velocity is zero) increases. Figs. 19–21 show the three dimensional graphs of function $f(\eta)$ for different values of stretching ratio parameter $A$.

Concluding remarks

The effects of nanofluid and stagnation point flow over a stretching cylinder are investigated numerically. Tangent hyperbolic fluid is considered as a base fluid. In order to check the endothermic and exothermic chemical reactions, heat generation/absorption effect is also included. A well-known technique (shooting method) is used to calculate the comparison, graphical and tabular results for the governing system of equations. In the light of present analysis following deductions may be drawn.

- A qualitatively different behavior was seen in the velocity profile for different values of stretching ratio parameter ($A > 1$ and $A < 1$).
- Thickening of the thermal boundary layer was observed for higher values of heat generation/absorption $\beta$.
- The thermal boundary layer thickness reduces for increasing values of Prandtl number $Pr$.
- Influence of thermophoresis parameter $N_t$ on nanoparticles concentration is similar as compared to the temperature.
- Velocity profile reduces for increasing values of Weissenberg number $\lambda$ and power law index $n$.
- The skin friction coefficient, local Nusselt number and local Sherwood number increases for increasing values of stretching ratio parameter $A$.
- Concentration profile reduces for large values of Lewis number Le but shows increasing behavior in the case of local Sherwood number.

References


