NUMERICAL INVESTIGATION OF TWO-DIMENSIONAL AND AXISYMMETRIC UNSTEADY FLOW BETWEEN PARALLEL PLATES

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Abstract In this study, heat and mass transfer in a viscous fluid which is squeezed between parallel plates is investigated numerically using the fourth-order Runge-Kutta method. The numerical investigation is carried out for different governing parameters namely; the squeeze number, Prandtl number, Eckert number, Schmidt number and the chemical reaction parameter. Results show that Nusselt number has direct relationship with Prandtl number and Eckert number but it has reverse relationship with the squeeze number. Also it can be found that Sherwood number increases as Schmidt number and chemical reaction parameter increases but it decreases with increases of the squeeze number.

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1. Introduction

The study of unsteady squeezing of a viscous incompressible fluid between two parallel plates in motion normal to their own surfaces independent of each other and arbitrary with respect to time has been regarded as one of the most important research topics due to its wide spectrum of scientific and engineering applications such as hydrodynamical machines, polymer processing, compression,
injection molding and lubrication system. The first work on the squeezing flow under lubrication approximation was reported by Stefan [1]. In 1886, Reynolds [2] obtained a solution for elliptic plates. The theoretical and experimental studies of squeezing flows have been conducted by many researchers [3–5]. The inadequacy of Reynolds equation in the analysis of porous thrust bearings and squeeze films involving high velocity has been demonstrated by Ishizawa [6]. Nanofluid heat transfer enhancement was studied by several authors [7–45].

Recently, due to their applications in many branches of science and engineering, the interest in the study of heat and mass transfer has been increased. Coincident heat and mass transfer with chemical reaction effect plays a vital role in design of chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing and cooling towers, distribution of temperature and moisture over grove fields, etc. Mahmood et al. [46] investigated the heat transfer characteristics in the squeezed flow over a porous surface. Abd-El Aziz [47] considered the outcome of time-dependent chemical reaction on the flow of a viscous fluid past an unsteady stretching sheet. Magneto-hydrodynamic squeezing flow of a viscous fluid between parallel disks was analyzed by Domairry and Aziz [48]. Most of engineering problems, especially some heat transfer equations are nonlinear, therefore some of them are solved using numerical solution and some are solved using the different analytic method, such as perturbation method, homotopy perturbation method, variational iteration method introduced by He. Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semi-exact methods which does not need small parameters is the Homotopy Perturbation Method. The homotopy perturbation method, proposed first by He in 1998 and was further developed and improved by He [49]. The method yields a very rapid convergence of the solution series in the most cases. The method yields a very rapid convergence of the solution series in the most cases. The HPM proved its capability to solve a large class of nonlinear problems efficiently, accurately and easily with approximations convergence very rapidly to solution. Usually, few iterations lead to high accuracy solution. This method is employed for many researches in engineering sciences. He’s homotopy perturbation method is applied to obtain approximate analytical solutions for the motion of a spherical particle in a plane couette flow by Jalal et al. [50]. Sheikholeslami et al. [51] studied rotating MHD viscous flow and heat transfer between Stretching and porous surfaces using HPM. They found that Increasing magnetic parameter or viscosity parameter lead to decreasing Nu while with increasing of rotation parameter, blowing velocity parameter and Pr the Nusselt number increases. In recent years some researchers used new methods to investigated flow and heat transfer characteristics [52–92].

In this paper, flow, heat and mass transfer in a viscous fluid which is squeezed between parallel plates is investigated numerically using the fourth-order Runge–Kutta method. Effects of active parameters on flow and heat transfer treatment are examined.

2. Mathematical formulation

We consider the heat and mass transfer analysis in the unsteady two-dimensional squeezing flow of an incompressible viscous fluid between the infinite parallel plates (Figure 1). The two plates are placed at $z = \pm \ell (1 - \alpha t)^{1/2} = \pm h(t)$. For $\alpha > 0$ the two plates are squeezed until they touch $t = 1/\alpha$ and for $\alpha < 0$ the two plates are separated. The viscous dissipation effect, the generation of heat due to friction caused by shear in the flow, is retained. This effect is quite important in the case when the fluid is largely viscous or flowing at a high speed. This behavior occurs at high Eckert number ($>1$).

Mass transfer with chemical reaction of the time dependent reaction rate is accounted. Further the symmetric nature of the flow is adopted.

The governing equations for mass, momentum, energy and mass transfer in unsteady two dimensional flow of a viscous fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right),$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\nu}{C_p} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right)^2,$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - K(t) C$$

$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - K(t) C$

Figure 1 Geometry of problem.

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Here \( u \) and \( v \) are the velocities in the \( x \) and \( y \) directions respectively, \( T \) is the temperature, \( C \) is the concentration, \( P \) is the pressure, \( \rho \) is the fluid density, \( \nu \) is the kinematic viscosity, \( k \) is the thermal conductivity, \( C_P \) is the specific heat, \( D \) is the diffusion coefficient of the diffusing species and \( k_1(t) = k_1(1-\alpha t) \) is the time-dependent reaction rate.

The relevant boundary conditions are:

\[
C = 0, \quad v = v_w = \frac{dh}{dt}, \quad T = T_H, \quad C = C_H \quad \text{at} \quad y = h(t),
\]

\[
v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = 0. \quad (6)
\]

We introduce these parameters:

\[
\eta = \frac{y}{\sqrt[3]{1-\alpha t}}, \quad u = \frac{\alpha x}{2(1-\alpha t)} f'(\eta),
\]

\[
v = -\frac{1}{2(1-\alpha t)^{1/2}} f(\eta), \quad \theta = \frac{T}{T_H},
\]

\[
\phi = \frac{C}{C_H}. \quad (7)
\]

Substituting the above variables into Eqs. (2) and (3) and then eliminating the pressure gradient from the resulting equations give:

\[
f'' - S(\eta f''' + 3f'' + f' f'' - f f'''') = 0, \quad (8)
\]

Using Eq. (7), Eqs. (4) and (5) reduce to the following differential equations:

\[
\theta'' + PrS(\theta' - \eta \theta') + PrEc (f' + 4\delta^2 f'^2) = 0, \quad (9)
\]

\[
\phi'' + ScS(\phi' - \eta \phi') - Sc\gamma \phi = 0, \quad (10)
\]

With these boundary conditions:

\[
f(0) = 0, \quad f'(0) = 0, \quad \theta'(0) = 0,
\]

We have:

**Table 1** Comparison of \(-\theta'(1)\) between the present results and analytical results obtained by Mustafa et al. [93] for viscous fluid \( S = 0.5 \) and \( \delta = 0.1 \).

<table>
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<th>( Ec )</th>
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<th>Present work</th>
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**Figure 2** (a) Effect of Prandtl number on the temperature profile when \( Ec = 0.5, S = 1 \); (b) effects of the squeeze number, Prandtl number on the Nusselt number profile when \( Ec = 0.5 \).

**Figure 3** (a) Effect of Eckert number on the temperature profile when \( Pr = 0.7, S = 1 \) and \( \delta = 0.1 \); (b) effect of the squeeze number and Eckert number on the Nusselt number when \( Pr = 0.7 \).

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$\phi'(0) = 0, \quad f(1) = 1, \quad f'(1) = 0,$
$\theta(1) = \phi(1) = 1,$ \hspace{1cm} (11)

where $S$ is the squeeze number, $Pr$ is the Prandtl number, $Ec$ is the Eckert number, $Sc$ is the Schmidt number and $\gamma$ is the chemical reaction parameter which are defined as:

$$S = \frac{\alpha^2}{2\nu}, \quad Pr = \frac{\mu C_p}{k}, \quad Ec = \frac{1}{C_p} \left( \frac{\alpha x}{2(1-\alpha t)} \right)^2,$$
$$Sc = \frac{\nu}{D}, \quad \gamma = \frac{k_1}{D}, \quad \delta = \frac{l}{x},$$ \hspace{1cm} (12)

Physical quantities of interest are the skin friction coefficient, Nusselt number and Sherwood number which are defined as:

$$Cf = \frac{\mu \left( \frac{\partial u}{\partial y} \right)_{y = h(t)}}{\rho u^2_w}, \quad Nu = \frac{-lk \left( \frac{\partial T}{\partial y} \right)_{y = h(t)}}{k T_H},$$
$$Sh = \frac{-lk \left( \frac{\partial C}{\partial y} \right)_{y = h(t)}}{DC_H}.$$

In terms of Eq. (7), we obtain

$$\frac{f^2}{x^2} (1-\alpha t) Re, C_f = f''(1),$$
$$\sqrt{1-\alpha t} Nu = -\theta'(1),$$
$$\sqrt{1-\alpha t} Sh = -\phi'(1).$$ \hspace{1cm} (14)

### 3. Results and discussion

The main objective of this study was to apply fourth-order Runge-Kutta method to obtain an explicit analytic solution of the heat and mass transfer characteristics in a viscous fluid which is squeezed between parallel plates. The present code is validated by comparing the obtained results with other works reported in Ref. [23]. As shown in Table 1, they are in a very good agreement.

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Figure 4 (a) Effect of Schmidt number on concentration profiles when $S = 1, \gamma = 1$; (b) effects of the squeeze number and Schmidt number on the Sherwood number when $\gamma = 1$.

Figure 5 (a) Effect of chemical reaction parameter on concentration profiles when $S = 1, Sc = 1$; (b) effects of the squeeze number and chemical reaction parameter on the Sherwood number when $Sc = 1$.
This study is completed by depicting the effects of the squeeze number, Prandtl number, Eckert number, Schmidt number and the chemical reaction parameter on the temperature and concentration profiles. Effect of Prandtl number on the temperature profile and Effects of the squeeze number, Prandtl number on the Nusselt number profile are shown in Figure 2. The small values of Prandtl number ($Pr < 1$) characterize liquid materials, which have high thermal diffusivity but low viscosity; however high-viscosity oils are represented by large values of Prandtl number ($Pr > 1$). The thermal boundary layer thickness decreases as Prandtl number increases. Therefore Nusselt number is increases function of Prandtl number. An increase in the squeeze number can be related with the decrease in the kinematic viscosity, an increase in the distance between the plates and an increase in the speed at which the plates move. As the squeeze number increase Nusselt number decreases and this effect is no significant for low values of Prandtl number.

Effect of Eckert number on the temperature and Nusselt number is shown in Figure 3. Increasing Eckert number leads to decrease thermal boundary layer thickness. So increase in Eckert number causes Nusselt number to increase. Effect of Schmidt number on concentration profiles and effects of the squeeze number and Schmidt number on the Sherwood number are shown in Figure 4.

When $Sc>1$, increasing Schmidt number leads to decrease concentration in centerline but opposite behavior is observed when $Sc<1$. Sherwood number increases with increase of Schmidt number. Figure 5 shows the Effect of chemical reaction parameter on concentration profiles and Sherwood number. It is worth mentioning here that $\gamma>0$ represents the destructive chemical reaction and $\gamma<0$ characterizes the generative chemical reaction. Concentration field is a decreasing function of destructive chemical reaction parameter and an increasing function of generative chemical reaction parameter.

4. Conclusion

In this study, heat and mass transfer in the unsteady squeezing flow between parallel plates is investigated numerically using the fourth-order Runge–Kutta method. Results show that Nusselt number has direct relationship with Prandtl number and Eckert number but it has reverse relationship with the squeeze number. Sherwood number is an increasing function of each of Schmidt number and chemical reaction parameter but it is decreasing function of the squeeze number.

References


