Credit risk contagion of Supply Chain Based on trade credit

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Abstract

There are two kinds of different credit risks in the two-echelon supply chain with typical capital constraints. One is the credit risk of the supplier to the bank loan, and the two is the credit risk which is formed by the supplier to the retailer with the commercial credit. Based on the newsboy model under the assumption of correlation and contagion between these two kinds of credit risk analysis. Research shows that: the two types of credit risk between the intensity of the infection and the supplier's production costs are positively correlated.

1. Introduction

Supply chain refers to the acquisition of raw materials, processed into semi-finished products until the product, and the product to the customer in the hands of some of the enterprises or sectors of the network(Wu Jun, Li Jian,2006)[1]. In a capital constraint in the supply chain, at the same time, there are two different types of credit risk, one is the supplier to the bank credit loans of credit risk; the other is the suppliers Allow the retailer delayed payments and the credit risk of the trade credit risk. Trade credit risk is a common supply chain in a class of credit risk, (once the retailer's default or credit risk increases), if not timely take preventive

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measures, the credit risk of the retailer will impact the suppliers, thus increasing the supplier's Bank credit risk. In other words, in the supply chain, because of the relationship between the supplier and the retailer, the risk of trade credit and bank credit risk is contagious. There are two main reasons for the infection of these two kinds of credit risks: one is the trade credit given by the supplier to the retailer, and the other is the strategic dependence among the supply chain enterprises. Trade credit risk will seriously affect the credit risk of the supplier itself. Therefore, the bank should consider the risk of credit risk in the supply chain when evaluating the supplier's credit risk. The characterization between enterprises in supply chain decision interdependence, illustrates the supply chain enterprise credit risk contagion mechanism, and to supply chain between affiliated enterprise credit risk contagion metrics, which will help banks to control the credit risk of the supply chain enterprises.

At present, the research on the credit risk contagion of affiliated enterprises, mostly concentrated in the enterprise group. Such as Chen Lin(2009,2010)[2-3] respectively in the structural model and the simplified model of two kinds of mode of enterprise group has the option to contact the parent company's credit risk contagion and between the two sub companies affected by the common parent company under the control of credit risk contagion of problems. Li Li(2015)[4] between the group and subsidiary company credit risk contagion mechanism was discussed. Analysis has been among the parent company and subsidiary company credit risk contagion intensity is path dependent, and with the increase in the proportion of equity increases gradually. Yang(2014) [5] Based on cooperative game theory to build the enterprise group associated with the basic transaction model, using the survival copula function describes the different types of related party transactions of the credit risk of the enterprise group, expounds the optimum allocation of resources can make the credit risk of the enterprise group the lowest level. In the study of the supply chain constraints, it is more focused on the supply chain coordination and efficiency and other issues (Chen,2008,Kouvelis2012,Lai,2009,Bing Jing,2014,Fu Yonghua,2014)[6-10]. In the research of the credit risk in the supply chain, most scholars have focused on the evaluation of the credit risk of small and medium enterprises in the supply chain(Bai Shizhen,2013,XiongXiong,2009,Hu Haiqing,2011, Kong Yuanyuan,2010)[11-14] As mentioned before, capital constraints in the supply chain in dominant suppliers in addition to its credit risk exists, there is also due to the retailer granted commercial credit to commercial credit risk, and credit risk will affect the supplier's own credit risk. At present, however, the commercial credit risk to the study on the supplier's own credit risk contagion is less, only Yang(Yang,2015) [15] Based on structural model by the commercial credit shock added to the diffusion process of jump of the commercial credit risk contagion. However the study not deterministic source is supplier asset value changes and ignore the influence of interaction between the supplier and the retailer of uncertainty. In conclusion, this paper analyzes the source of the supply chain between enterprises of the specific interaction and commercial credit supply chain credit risk contagion problem, trying to supply chain credit risk of enterprises to provide some ideas for commercial banks to control.
2. The contagion of credit risk among supply chain enterprises

2.1 Problem description

Consider a typical two-level supply chain consisting of one supplier and one retailer. Among them, the supplier is the supply chain leader and the retailer faces the newsboy market demand. For the convenience of discussion, we assume that funds under the constraint of supply chain enterprise credit risk contagion, without loss of generality, the assumption free funds of the supplier and the retailer are 0, ordering decision is described as a dominant supplier Stackelberg game. In the core status of suppliers, through a bank loan to obtain funds to organize production and retailers by its own limitations, unable to get funds from the bank, can only rely on suppliers to provide the commercial credit, take delay to pay to obtain financing demand. Draw on the work of Kouvelis & Zhao[13], assuming that banks are in a fully competitive market. The other variables involved in the model are shown below:

- \( r_s \): Retailer business credit cost;
- \( \xi \): Market demand for commodities, the probability density function, \( f_\xi \), the probability distribution function \( F_\xi \), assuming that the uniform distribution in \([0, B]\). The distribution of demand for retailers and suppliers is public information;
- \( \omega \): Commodity wholesale price;
- \( q \): Order quantity of goods;
- \( p \): Commodity market prices, may wish to assume \( p \geq w \);
- \( c \): The cost of unit product suppliers, assuming the products for perishable goods, merchandise sales in units of the final unsold residual value is 0;
- \( \pi_r \): Retailer's profit;
- \( \pi_s \): Supplier's profit;
- \( r_f \): Risk free interest rate;
- \( r_b \): Interest rates on bank loans to suppliers;
- \( k_r \): When the market demand of the retailer default threshold;
- \( k_s \): Supplier market demand default threshold.

2.2 Model Assumptions

Sales at the beginning of the period, funding constraints of retailers to internal financing modes of commercial credit obtained financing service, and the order \( q \) to the supplier and payment \( wz \) in the form of accept suppliers to provide wholesale contracts, which are then retailers at a fixed price \( p \) in the market for retail sale \( \min \xi, q \). Also subject to funding constraints of suppliers in accordance with the wholesale contract, the capital market in the competition to get the \( cq \) scale of financing services, the interest rate for the \( r_b \). At the end of the sales cycle, the retailer will return the money to the supplier. After receipt, the supplier
will return the money to the repayment of principal and interest of bank loans. At the end of the sales cycle, the supplier may from a retailer get principal and interest \( \min p \min \xi, q \), when demand is insufficient \( pD < wq \), the retailer’s sales income is not sufficient to repay the suppliers, retailers will default supplier to suffer a loss. At this time, if a serious lack of market demand, supplier from retailers to recover payment insufficient to repay the Bank of principal and interest \( pD < cq(1 + r_b) \), supplier default. On the contrary, the bank can obtain the expected return.

2.3 Optimal decision making of enterprises in supply chain

According to the previous assumption, the retailer’s profit function can be obtained.

\[
\pi_r = E[p \min (D, q) - wq(1 + r_b)]^+ - \frac{1}{2}(1 - \alpha)e^2
\]

\( k_r = \frac{wq}{p} \) is the minimum requirement for a retailer when it goes bankrupt, then (1) can be reduced to:

\[
\pi_s = E[p \min (D, q) - p \min (D, k_r)]^+
\]

If \( k_r > q \) the retailer doesn’t borrow, so suppose \( 0 < k_r < q \). On type can be changed into

\[
\pi_s = pE[\min (D, q)] - pE[\min (D, k_r)]
\]

Supplier’s profit function:

\[
\pi_s = E[N(\xi)] - cq(1 + r_f) - \frac{1}{2}\alpha e^2
\]

Among them:

\[
E[N(\xi)] = E[\min \{ p \min (D, q), wq \}] = E[p \min (D, q)] - \pi_r^{(s)} - \frac{1}{2}(1 - \alpha)e^2
\]

Because the bank is in a competitive equilibrium, so there is

\[
cq(1 + r_f) = E[\min \{ p \ min (D, q), cq(1 + r_f) \}]
\]

So can get

\[
\pi_s = E[p \ \min (D, k_r)] - 1; q(1 + r_f) - \frac{1}{2}\alpha e^2
\]

**Proposition 1:** In the supplier’s business credit model, the retailer’s optimal level of effort and order quantity

\( e^* = 0, q^* = \frac{B}{2} \); the supplier’s optimal wholesale price and share coefficient

\( w^* = 1, \alpha^* = \frac{B^2 - 2Bc - B - 2c + 2}{B(-2c + B - 2c - 2)} \).

**Proof:** The retailer’s profit function is:

\[
\pi_r^{(s)} = E[p \min (\xi, D) - p \min (\xi, k_r)]^+ = E[p \min (\xi, D)] - E[p \min (\xi, k_r)]
\]
According to the first order condition:

\[
\frac{\partial \pi_r^{(s)}}{\partial q} = 0, \quad \frac{\bar{F}(q-e)}{\bar{F}(k_r-e)} = \frac{w}{p} = 1
\]

\[
\frac{\partial \pi_r^{(s)}}{\partial e} = 0, \quad p \cdot F(q-e) - p \cdot F(k-e) = e \cdot (1 - \alpha)
\]

Can get:

\[
e = \frac{B(w-1)}{B\alpha w + B\alpha - Bw - B - w} + 1, \quad q = \frac{B^2 \alpha - 1}{B\alpha w + B\alpha - Bw - B - w} + 1
\]

And by (4) the supplier’s profit function:

\[
\pi_s = E[p \min(D, k_r)] - cq(1 + r_f) - \frac{1}{2} \alpha e^2
\]

\[
\frac{d \pi_s}{dw} > 0, \text{the optimal wholesale price of the supplier } w^* = p = 1,
\]

\[
\alpha = \frac{B^2 - 2Bc - B - 2c + 2}{B(B - 2c - 2)} \text{, then, } e = 0, q = \frac{B}{2}.
\]

And because of \( cq = pk - p \int_{\xi_r}^{k_r} F(\xi)d\xi \), the following conclusions can be obtained:

**Proposition 3:** In the internal financing mode, the minimum requirement of the supplier’s bankruptcy:

\[
k_s = e + B - \sqrt{-2Bcq + B^2 + 2Be} = B - B\sqrt{1 - c}
\]

3. **commercial credit risk to the bank credit risk transfer**

Credit risk refers to the possibility that the borrower fails to repay the debt or bank loan in time and in full in all kinds of reasons[14]. Just as the previous analysis, the credit risk of the supplier in this paper is produced by the retailer's commercial credit risk contagion, and the measurement of the intensity of infection will be the focus of the research.

In order to facilitate the analysis, in this, a random variable \( z_r \) is defined to represent the retailer event of default. The value of \( z_r = 0 \) or 1, \( z_r = 1 \) said the retailer defaults, \( z_r = 0 \) said the retailer did not breach. Similarly, a random variable \( z_s \) is defined to represent the supplier's default event. The value of \( z_s = 0 \) or 1, \( z_s = 1 \) means that the supplier is in breach of contract, \( z_s = 0 \) indicates that the supplier is not in breach of contract.

From the results of 2.2, we can know that the optimal order quantity of the retailer is \( q^* = \frac{B}{2} \), the wholesale price of the supplier is \( w^* = 1 \). Then

\[
k_r = \frac{B}{2}
\]

Supplier bankruptcy threshold:
By (8) it can be seen that the supplier's default probability is influenced by the cost of production, the level of retailer's effort and the market demand. Obviously, when the market price is certain, the increase in production costs will reduce the profit of the supplier and therefore increase the probability of its default. And the impact of the level of effort, the reason is because the level of effort will affect the market demand. On the one hand, efforts will increase the market demand, on the other hand, retailers strive to pay the cost of demand. So, for retailers, how to determine the optimal level of effort is also very important. And for the supplier, how to stimulate the retailer to improve the level of effort is also very important, so choose to share the cost of effort. However, it is not advisable for retailers and suppliers to increase market demand to make up for the cost of effort. And the impact of market demand on the supplier's credit risk is transmitted through the retailers. Retailers face the commodity market directly, if it is too optimistic about the market estimate, resulting in its order quantity increases, which will lead to an increase in the likelihood of retail defaults. If the retailer defaults, it will increase the likelihood of default by the supplier. In other words: retailers are facing the risk of market transformation for it to suppliers of commercial credit risk, and further through related party transactions evolution for suppliers of bank credit risk, the realization of the two different risk through the supply chain and infectious and evolution. So far, the supply chain credit risk in the face of market demand when the random newsboy model of infection.

In the supply chain, the credit risk caused by market fluctuations, the commercial credit as a link, from retailers to suppliers. In order to quantitatively describe the intensity of infection, define the:

**Definition:** The conditional probability of supplier's default when the retailer breaches the contract is the relationship between the supply chain and the credit risk $\lambda$.

The following proposition can be obtained by defining a combination (4) and (5):

**Proposition 4:** Retailer commercial credit risk to supplier credit risk contagion intensity:

$$\hat{\lambda} = p(Z_s = 1 | Z_r = 1) = 2 - 2\sqrt{1 - c}$$

It can be seen that the intensity of the credit risk contagion in the supply chain is affected by the market demand and the supplier's production cost.

4. **Sensitivity analysis of credit risk contagion intensity**

Retailer commercial credit risk to supplier credit risk contagion intensity:

$$\hat{\lambda} = p(Z_s = 1 | Z_r = 1) = 2 - 2\sqrt{1 - c}$$

After a simple mathematical operation, we can get the following conclusions:

**Proposition 5:** When market demand is subject to uniform distribution in $0, B$, the supplier and retailer credit risk contagion intensity has nothing to do with the market demand parameter $B$, and it is an increasing function of the supplier's production cost.
As can be seen from the above, the strength of the associated credit risk increases with the increase in the cost of the supplier. The reason, the retailer's profit is always 0, and with the increase in production costs, the supplier's profits will be reduced, so as to increase the possibility of supplier bankruptcy, and thus increase the strength of the associated credit risk.

5. Conclusion and Prospect

Commercial credit is a very common internal financing model, but it is very easy to cause the retailer's commercial credit risk to the supplier's credit risk. This paper in the supplier to external loans of financial institutions and to give retail credit of evolution mechanism of the supplier and the retailer credit risk contagion, to measure the credit risk of the intensity of infection, and further discusses the influence of supplier cost of supplier and retailer credit risk contagion intensity. It is found that the credit risk contagion intensity is a function of the supplier's production cost.

In order to describe the supply chain credit risk contagion, this paper makes the necessary simplification in the model processing. However in the real world, credit risk contagion causes not only contains the credit, debit and credit relationship, such as the external environment of the enterprise management change; and in actual operation in an enterprise may exist in the supply chain, so the supply chain of the chain there will be credit risk contagion. To consider the relationship between multiple enterprises credit risk will be the next step in the study of the problem. This paper on supply chain credit risk contagion of tentative research. On some of these issues are necessary for abstract simplification, thus have some assumptions and the actual may be, there are some gaps, such as free enterprise funds are 0, the commodity market obey uniform distribution, banks are in perfect competition market and risk neutral and. Our next step will further relax these assumptions, making the study more consistent with the actual.
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Reference