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Superconducting antenna concept for gravitational waves

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Abstract

The most advanced contemporary efforts and concepts for registering gravitational waves are focused on measuring tiny deviations in large arm (kilometers in case of LIGO and thousands of kilometers in case of LISA) interferometers via photons. In this report we discuss a concept for the detection of gravitational waves using an antenna comprised of superconducting electrons (Cooper pairs) moving in an ionic lattice. The major challenge in this approach is that the tidal action of the gravitational waves is extremely weak compared with electromagnetic forces. Any motion caused by gravitational waves, which violates charge neutrality, will be impeded by Coulomb forces acting on the charge carriers (Coulomb blockade) in metals, as well as in superconductors. We discuss a design, which avoids the effects of Coulomb blockade. It exploits two different superconducting materials used in a form of thin wires -"spaghetti." The spaghetti will have a diameter comparable to the London penetration depth, and length of about 1-10 meters. To achieve competitive sensitivity, the antenna would require billions of spaghettis, which calls for a challenging manufacturing technology. If successfully materialized, the response of the antenna to the known highly periodic sources of gravitational radiation, such as the Pulsar in Crab Nebula will result in an output current, detectable by superconducting electronics. The antenna will require deep (0.3K) cryogenic cooling and magnetic shielding. This design may be a viable successor to LISA and LIGO concepts, having the prospect of higher sensitivity, much smaller size and directional selectivity. This concept of compact antenna may benefit also terrestrial gradiometry.

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1. Introduction

Acceleration of conductors, as has been brilliantly demonstrated by Tolman and Stewart (TS) (1916) may generate a current in closed circuits. Can a gravity field, which is locally equivalent to an acceleration, cause a similar effect? This question is not as simple as it looks at first glance. Indeed, in the TS case the acceleration is mechanically delivered to the ionic lattice only, while in the case of a gravity field both electrons and ions would be equally accelerated and no current will result. A further difficulty is that in terrestrial laboratories, the gravity fields are locally homogeneous, while in the case of the circularly accelerated coils used by TS, the inertial forces are tangential to the circumferential conductors. For tangential acceleration of charge carriers by gravity forces, one can consider a linear bar along the force line. In a "frozen lattice" model (where some hypothetical external force neutralizes the gravitational forces on the ions) only electrons will move. However, in finite length conductors this motion will be stopped by charge accumulation at its ends ("Coulomb blockade"), and an electric field $E_{TS}=-mg/e$ will be generated in the conductor. Here g is the acceleration, m is the rest mass, and e is the (negative) charge of the electron. The field E_{TS} opposes the motion of electrons under gravity. Real lattices are not "frozen". Indeed, the crystal is stabilized by Coulomb interaction and quantum exchange forces and no other forces are involved. Since gravity accelerates all the constituent masses, for the lattice not to move, it should be hanged on or leaning against a rigid surface. It was shown, however, that in this situation *E_{TS} is negligible,* and the dominating electric field has an opposite sign, so that in the laboratory electrons will move opposite to the direction of g! This feature was first pointed out by Dessler *et al*. (1968), confirmed experimentally by Beams (1968), and accepted by the community. As summarized by Kogan (1971), the electric field acting on electrons can be stronger than the E_{TS} by a factor of M/m, where M is the ionic mass. The total electric field is therefore

$$E = \alpha Mg / e - mg / e \approx \alpha Mg / e . \qquad (1)$$

The dimensionless coefficient α depends on the parameters of the material, and is estimated to be $\alpha \sim 0.1-1$. This is further elaborated in Appendix A.

Gravitational waves accelerate masses by tidal quadrupolar gravitational forces, which are symmetric relative to the centre of mass of the system (Fig.1, *a* and *b*). The cross-bar antenna shown in Fig. 1c,in principle, may serve as a simple GW-detector, responding with AC-current through the bridge (see Fig. 1*c*) to the action of induced quadrupolar deformation. However, as will be discussed later, its sensitivity falls short of detectability because gravity forces are far weaker than the electric forces which they should balance. The closed circuit antenna which we discuss now is free of this drawback.

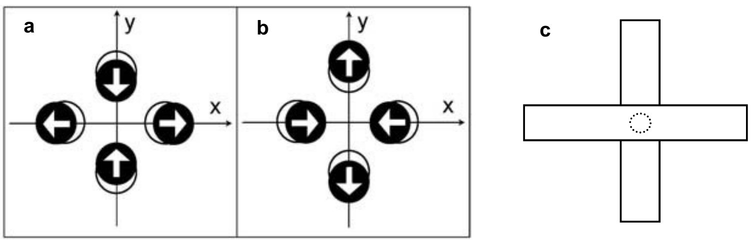

Fig. 1. (a) and (b): quadrupolar acceleration induced in two half-periods of "+" polarized GW impinging on particles;(c) two bars in the GW field are deformed opposite to each other: electron redistribution should create a current along a bridge (dotted lines, face view) connecting them.

2. Basic principle

Proposed antenna geometry is shown in Fig.2. The force acting on a lattice ion at a distance x from the centre of mass [see, *e.g*., Adler (1976)] is:

$$F(x) = M\ddot{h}x / 2 ; \; \ddot{h} \equiv d^2 h / dt^2 \qquad (2)$$

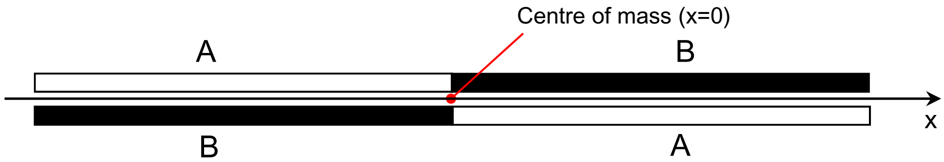

Fig.2. The GW $h=h_0 Sin\omega t$ is incident perpendicularly to the plane of the two bimetallic conductors. Each conductor, A or B, is of length L. The ends of the conductors are connected (not shown) to close the loop.

where the dimensionless amplitude h of the GW is defined via induced small coordinate deviation $\Delta x = xh/2$. Then the analogy of Eq. (1) for the case of GW is

$$E(x) = \alpha M\ddot{h}x/2e \tag{3}$$

The amplitude of the electromotive force $\mathcal{E}$ is the integral of the E-field over the closed loop [Landau and Lifshitz (1960), Feynman *et al.* (1964)]:

$$\mathcal{E} = \oint_{A-B-A-B} E(x)dx = -(\ddot{h}L^2/2e)(\alpha_A M_A - \alpha_B M_B) \tag{4}$$

In writing this we neglect the contribution of the connectors between two bimetallic wires in Fig. 2.

A time-periodic electric field $E = E_0 \exp(i\omega t)$ generates the following total current density in a superconductor [Van Duzer (1998), Tinkham (1975)]:

$$j = j_n + j_s = (\sigma_1 - i\sigma_2)E \tag{5}$$

where

$$\sigma_1 = (n_n e^2 \tau)/[m(1+\omega^2\tau^2)],\ \sigma_2 = n_s e^2/(m\omega) + n_n e^2(\omega\tau)^2/[m\omega(1+\omega^2\tau^2)] \tag{6}$$

In Eq. (6), n_s and n_n are superconducting and normal electron densities, ω is the frequency, and τ is the relaxation time of the normal electrons (typically, $\tau{\sim}10^{-13}$ s). The smallness of $\omega\tau$ allows keeping only the first term in σ_2 of Eqs. (6) with the result

$$j \approx j_s = -i\frac{n_s e^2}{m\omega}E\,. \tag{7}$$

The requirement of having the same current flow through all parts of our circuit implies that the electric field E in (7) is also spatially constant. For the harmonic GW mentioned in the caption to Fig. 2 with amplitude h_0 and frequency ω, the current amplitude is:

$$I = |jS| = [n_s e\omega h_0 LS/(8m)]|\alpha_A M_A - \alpha_B M_B|\,. \tag{8}$$

This expression is immediately applicable to periodic sources, such as the Crab Pulsar.

3. Inductance and magnetic energy

In the above calculations we neglected the magnetic inductance effects. To justify such a procedure the magnetic energy: $E_{mag} = \mathcal{L}I^2/2$ (where $\mathcal{L}$ is the circuit inductance) should not exceed the kinetic energy associated with the current flow: $E_{kin} = LSn_s m\text{v}^2$. Otherwise, the energy would be transferred to the magnetic field rather than to the charge motion which causes it. For a single loop in Fig.2 their ratio is

$$E_{mag} / E_{kin} \sim \mu_0 L e^2 n_s{}^2 \text{v}^2 S^2 / LSn_s m\text{v}^2 = \mu_0 e^2 n_s S / m \tag{9}$$

(where $\mu_0 = 4\pi \cdot 10^{-7}$ *Henry/meter*). Substituting $n_s \sim 10^{22} cm^{-3}$ into Eq. (9), we find that $E_{mag} / E_{kin} \sim 1$ when $S \sim 10^{-14} m^2$, *i.e.*, the wire diameter is about 0.1μm. This choice has the extra advantage of using superconducting materials most effectively, since the diameter of the wire is close to the London penetration depth λ of typical superconductors. Substituting S and $[(\alpha M / m)_A - (\alpha M / m)_B] \sim 10^4$ (which one can expect, say, for vanadium-lead bimetallic pair) into Eq. (9), we find $I \approx 2 \cdot 10^{-2} (\omega / Hz) h_0$. Thus, for $h_0 \sim 10^{-26}$ and $\omega \sim 2\pi(60)Hz$ we have $I \approx 0.7 \cdot 10^{-25}$ Amp in a single loop. In principle, one can imagine a design made of a large number N of bimetallic wires with currents in opposite direction in neighbouring conductors. The wires can be merged into a single read-out circuit, as shown in Fig. 3.One can, in principle, choose N as large as required to make the current detectable.

4. Sensitivity

The sensitivity of the antenna is determined by the signal-to-noise ratio. The unavoidable component of the noise is the Johnson-Nyquist noise associated with the normal component (*i.e.*, unpaired electrons): $\langle I_{noise} \rangle = [4(k_B T / R_n)\delta\nu]^{1/2}$ (Van Duzer (1998), Tinkham (1975)), where k_B is the Boltzmann constant, R_n is the resistance of the normal component of the superconductor, and $\delta\nu$ is the measurement bandwidth. The detection of the GW radiation at unknown frequency is only possible if $I_{signal}/I_{noise} > 1$, *i.e.*,

$$I_{signal} / I_{noise} \sim en_s SL\omega(\alpha M / m) h_0 / \{8[4(k_B T / R_n)\delta\nu]^{1/2}\} > 1 \tag{10}$$

This estimate is for a single loop. For N loops, the signal will scale linearly with N, and the noise will scale as $N^{1/2}$; thus signal to noise will scale as $N^{1/2}$. That means the amplitude h_0 can be no smaller than $h_0 \approx 16[(k_B T / R_n)\delta\nu]^{1/2} / N^{1/2} en_s SL\omega(\alpha M / m)$. Since $R_n = (\rho L / S)\exp(\Delta / k_B T)$, where ρ is resistivity of the material above the transition temperature T_c, and $\Delta = \Delta(T)$ is the BCS gap in the spectrum of single-electron excitations [Van Duzer (1998), Tinkham (1975)], at $T << T_c$, $2\Delta(0) \approx 3.52 k_B T_c$:

$$h_0 \approx [16(k_B T\delta\nu)^{1/2} / en_s S^{1/2} L^{3/2} \rho^{1/2} \omega(\alpha M / m) N^{1/2}]\exp(-0.88 T_c / T) \tag{11}$$

At the parameters $L \sim 1m$, $S \sim 10^{-14} m^2$, $n_s \sim 10^{28} m^{-3}$, $\omega \sim 2\pi(60)Hz$, $\alpha M/m \sim 10^4$, $\rho \sim 0.1 m\Omega\text{-}cm$, $T \sim 0.3K$, and $N \sim 10^{12}$: $h_0 \approx 0.5 \cdot 10^{-22} (\delta\nu / Hz)^{1/2} \exp(-0.88 T_c / T)$. Since $T_c \geq 5K$ (for the V-Pb bimetallic pair), at $T_{op} \sim 0.3K$ the exponential factor is $\sim 4 \cdot 10^{-7}$, and thus $h_0 \approx 2 \cdot 10^{-29} (\delta\nu / Hz)^{1/2}$, making the Crab Pulsar GW radiation detectable.

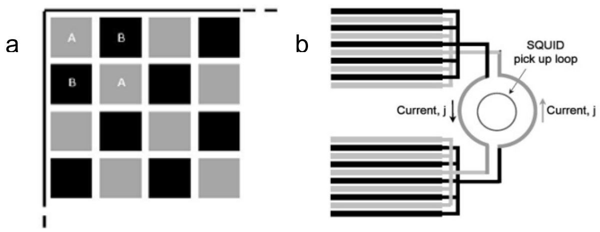

Fig. 3. (a)-cross sectional view of wire arrangement; (b)-schematics of multi-wire interconnections for non-invasive current measurement via SQUID. The current in materials *A* and *B* flows in opposite directions: two semi-circles constitute a full current loop for inductive pick up.

5. Discussion and conclusions

To summarize, a concept for a superconducting GW detector is proposed. The detector transforms part of the GW energy into the motion of superfluid electrons which is then detected electronically. The technical realization of the antenna may require some technological efforts, but there is no showstopper. For example, each "spaghetti" should have $\sim 0.1\mu m$ diameter. With the number of spaghettis$N\sim 10^{12}$, the total cross-section of the current loop is $\sim 10^{-2}m^2$. With $\sim 1Hz$bandpass filter this will allow direct detection of the Crab Pulsar, since the signal-to noise will be >1. It is interesting to note, that at $h \sim 10^{-26}$ this pulsar spreads on the Earth's orbit a GW radiation with the energy flux density $w = (c^3/16\pi G)\omega^2 h^2 \sim 10^{-10} ergs \cdot cm^{-2} s^{-1}$, where $G \approx 7\cdot 10^{-11} N(m/kg)^2$ is Newton's constant [Blair *et al.* (2012)]. It corresponds to ellipticity of this radiating system $\varepsilon \approx w^{1/2}$ [Weinberg (1972)], which turns out to be $\sim 10^{-5}$. The geometrical cross section of our antenna is $10^4 cm^2$, and per a half-period, the GW energy available for absorption is $\sim 10^{-8}$ *ergs*. This may be compared with the kinetic energy of electrons $E_{kin} = LmS^2 n_s^{\,2} e^2 v^2 / e^2 n_s S = I^2 Lm / e^2 n_s S = 10^{-46} J = 10^{-39} ergs$, indicating that efficiency of energy conversion in our antenna $\sim 10^{-31}$.

When detecting sources whose frequency is known with high accuracy, one can use the lock-in amplification, and detect signals much smaller than the noise level. At the chosen number of wires *N*, the value of the current signal to be detected is $I_{signal} \sim 10^{-13} A$, which, in principle, can be readily done. This is the benefit of a closed superconducting loop design (Fig. 2). By contrast, the action of GW with the same parameters for the cross-bar antenna (Fig. 1c) would generate a voltage across the bridge: $V = \delta\mu / e \sim \alpha M h \omega^2 L^2 / e \sim 10^{-26} Volt$ for an antenna size $L=10m$[1]. This voltage will cause a charge transfer of $\delta q = VC$ between the bars. However, δq is far smaller than a single electron charge for any reasonable value of capacity C in the circuit, and thus is undetectable.

The relatively large value of I_{signal} in the adopted closed circuit case leads to the possibility of reduction in the number of spaghettis or/and reduction in the size of the antenna. That makes the device easier to fabricate. The fabrication of the device will require thin film deposition and nanolithography methods. Another possibility is the eventual use of superconducting nanotubes as spaghettis. The estimates we provide above demonstrate that very high sensitivity could be obtained with these devices. They are non-resonant and may be used for a wide variety of sources. Hopefully, in parallel with other recognized efforts [Blair *et al.* (2012)], the suggested superconducting device will be realized in a most challenging physics measurement – the detection of gravitational waves.

Appendix A. Upwards motion of electrons in Earth's gravity field

Suppose we have a bar with length L (Fig. A1), consisting of atomic layers separated by a distance a with total number of layers $N = L/a$. Suppose there is no *g*-field initially. Let the bar be compressed along its

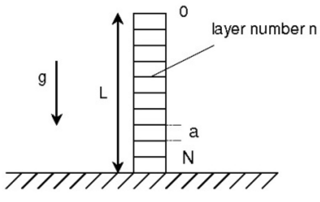

FIG.A1. Metallic bar supported by a rigid platform. Atomic layers from 0 to N are shown. N>>1.

length L by a force F, so that its length is reduced by $\Delta L = F/k$. Then the variation of the initial density of ions and

[1]Adler (1976) suggested using two different metallic bars, aligned in parallel, for GW detection and obtained similarly small value for V.

electrons is (we consider one conductivity electron per ion): $\delta n / n = \Delta L / L = F / kL$. Since the Fermi-energy $E_F \propto n^{2/3}$, and the rigidity k is related to the Young modulus Y by $k = YS / L$, where S is the cross-section of the bar, then

$$\frac{\delta E_F}{E_F} = \frac{2}{3}\frac{\delta n}{n} = \frac{2F}{3kL} = \frac{2F}{3YS} = \frac{2}{3}\frac{p}{Y}, \quad \text{(A1)}$$

where $p = F / S$ is the applied pressure. In the gravity field, the pressure in the rod is due to its own weight. The force acting on the bottom layer is $F = M(N-1)(S/a^2)g \approx MNSg/a^2$. This creates a shift of the chemical potential (we consider the case of *T=0K,* so that $\mu = E_F$):

$$\frac{\delta\mu}{\mu} \sim \frac{p}{Y} \sim \frac{N(S/a^2)Mg}{SY} \sim \frac{NaMg}{Ya^2 a} = \frac{MgL}{Ya^3} \sim \frac{MgLa}{e^2} \sim \frac{MgL}{E_F}, \quad \text{(A2)}$$

which means that the shift of chemical potential at the bottom is proportional to the ratio of the gravitational potential energy of *ions* to the Fermi energy of *electrons*. In writing Eq. (A2) we made use of the fact that $Y \approx e^2 / a^4$. Indeed, the deformation energy associated with two neighbouring atomic layers is $W_a = k_a(\delta a)^2 / 2$, where $k_a = Nk$ (the shorter the length element, the stronger the rigidity). If the layers are compressed by $\delta a \sim a$, then $W_a \sim (S/a^2)E_F \sim Se^2/a^3$. Therefore, $k_a \sim Se^2/a^5$, and $k \sim Se^2/Na^5 = Se^2/La^4$, thus yielding the above estimate for Y.

Since at the top layer the pressure is zero, the gravitational acceleration g creates a relative shift of chemical potential between the top and bottom, which is obviously a linear function of the vertical coordinate. Its gradient is directed opposite to the direction of g and equals

$$\nabla(\delta\mu) = \frac{\delta\mu}{L} = -\alpha Mg, \quad \text{(A3)}$$

where the dimensionless parameter α is inserted to correct the above approximations. To minimize their total energy, the electrons will move from the region of with higher μ to regions of smaller μ. In Fig. A1 they will move upwards and build up a negative charge at the top layer of the rod, and leave the bottom positive. An electric field $E = \alpha Mg$ will arise then because of this charge redistribution, neutralizing eventually the action of $\nabla\mu$.

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