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Numerical prediction of wave-making resistance of pentamaran in unbounded water using a surface panel method

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Abstract

This paper investigates the analysis of potential flow around the pentamaran hull moving with a uniform velocity in unbounded water using a surface panel method. The free surface boundary condition is linearized by the systematic method of perturbation in terms of a small parameter. The surfaces are discretized into flat quadrilateral elements and the influence coefficients are calculated by Morino's analytical formula. Dawson's upstream finite difference operator is used in order to satisfy the radiation condition. The pressure Kutta condition is imposed at the trailing edge of the lifting body to determine the dipole distribution, which generates required circulation on the lifting part. The effect of the hull stagger on the wave making resistance of the pentamaran hull is analyzed.

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Keywords: Resistance; panel method; Kelvin free surface condition; pressure kutta condition; pentamaran

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Nomenclature

C_W	wave- making coefficient
g	acceleration due to gravity
K_0	wave number
n	unit normal vector on the surface
S_F	free surface
S_H	hull surface
S_∞	surface of a hemisphere
U	uniform velocity in the positive x-direction
Greek symbols	
Φ	total velocity potential
ε	perturbation parameter
ϕ	perturbation velocity potential due to uniform flow
ϕ_1	first order perturbation velocity potential
ϕ_2	second order contributory part for perturbation velocity potential
ζ	wave elevation
ζ_1	first order wave elevation
ζ_2	second order contributory part for wave elevation

1. Introduction

The increasing demand for fast sea transportation has led till now to a significant growth of interest in multi-hull ships for coastal areas, where at present the catamaran seems the leading commercial type. Recently new unconventional larger

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multi hull crafts have been proposed, the study and the development of such hull-forms is aimed both at reducing the high fuel consumption inevitably linked to the higher speeds of monohull ships and at achieving the advantages of larger deck areas.

The pentamaran seems interesting possibilities due to the benefits given by their very slender hull forms compared to the conventional multi hull ships at various medium-high speeds. However, except catamaran, trimaran ships have had a limited use till now and there is very little information about pentamaran hull forms and configurations. Besides, it is necessary a comparison of the hydrodynamic characteristic for the various configurations of pentamaran hull as well as the other different high speed vessels.

The multihull vessel is composed of main hull with two or more small outriggers and has a wide range of choices for reducing the wave-making resistance by exploiting arrangements of the hull elements and varying hull forms. This multihull resistance presents a complex phenomenon for the ship designer as the interference effects must be considered in addition to the resistance of the demihulls in isolation.

In the field of ship hydrodynamics, the first theoretical solution for the problem of wave resistance was given by Michell [1] for a thin ship moving on the surface of an inviscid fluid. The solution is the well-known Michell Integral based on the double Fourier transformation of the velocity potential. Later on Kelvin [2] established the fundamental theory of ship waves. Since then many theoretical studies in ship hydrodynamics have been undertaken. Havelock [3] studied the effects of shallow water on the wave resistance and wave pattern for a point pressure impulse traveling over a free surface.

Peng [4] presented a numerical method based on Michell's theory for the resistance in calm water and for the sea keeping performance in waves of multihull of various configurations at an infinite depth of water. The problem is solved by the boundary element method in terms of the Green function.

Moraes et al [5] investigated the wave resistance component of high speed catamarans and the effects of shallow water on these wave resistance components. Two methods were applied: the slender body theory proposed by Michell and a 3D method used by Shipflow software. Results were obtained for different types of twin hulls and attention was given to the effects of catamaran hull spacing.

Yeung [6] provided a general expression for calculating the interference resistance of multihull in a simple integral. This expression contains the explicit effects of stagger and separation in where it requires only the knowledge of the Kochin functions of each of the interacting hulls.

Begovic et al. [7] carried out an experimental research at Naples University towing tank by model test of a pentamaran hull for a ship to be used on medium distance Mediterranean routes. The main hull was slender round bilge hull forms from Series 64 and the very slender outrigger hull forms are also from Series 64. The isolated main hull, the isolated outriggers and the whole pentamaran hull in different configurations were tested in calm water in order to evaluate the hydrodynamic characteristics and powering performances.

Tarafder and Suzuki [8] investigated the influence of the water depth and the wave interference effects on the first and second order wave making resistance of the catamaran hull using a potential based boundary element method. The effects of hull separation and water depth on the hydrodynamic characteristics of the catamaran hull were analyzed and the validity of the computer scheme was examined by comparing the wave resistance with others' numerical results.

Wang [9] developed a 3D Rankine source panel method for calculating the linear wave-making resistance of a trimaran with Wigley hulls. Non uniform rational B-spline (NURBS) was adopted to represent body surface and to calculate the normal vector and derivative vectors of the NURBS surface. The radiation condition was satisfied using the numerical technique of staggered grids.

Aubault and Yeung [10] provided a brief description of a formulation for the interference wave resistance between two or more hulls in finite-depth waters, using a distribution of Havelock sources over the hulls. The formulation was based on thin ship approximation.

The aim of the present paper is to draw a mathematical model for computing the potential flow around pentamaran hulls moving with a uniform speed in unbounded water and to compute the wave-making resistance, wave profile and wave pattern at various speeds.

2. Mathematical modeling of the problem

Consider a ship moving with a constant speed U in the direction of the negative x-axis. The z-axis is vertically upwards and the y-axis extends to starboard. The origin of the co-ordinate system is located in an undisturbed free surface at amidship, so that the undisturbed incident flow appears to be a streaming flow in the positive-x direction. The fluid is assumed to be incompressible and inviscid where the flow is irrotational. The velocity potential Φ and wave elevation ζ can be expressed as

$$\begin{aligned}\Phi &= Ux + \phi = Ux + \sum_{n=1}^{\infty} \varepsilon^n \phi_n \\ \zeta &= \sum_{n=1}^{\infty} \varepsilon^n \zeta_n\end{aligned} \tag{1}$$

where, ε (ε<<1) is a perturbation parameter and is defined by B/L. B and L are the breadth and the length of the ship respectively. ϕ is the perturbation velocity potential due to the existence of the hull. The velocity potential Φ satisfies the Laplace equation

$$\begin{aligned}&\nabla^2 \Phi = 0 \\ &\nabla^2 (Ux + \phi) = 0 \ \text{ in the fluid domain V} \\ &\nabla^2 \phi = 0\end{aligned} \tag{2}$$

The fluid domain V is bounded by the hull surface S_H, free surface S_F and the surface of a hemisphere S_∞ of infinite radius. Now the problem can be constructed by specifying the following boundary conditions as follows:

(i) *Hull boundary condition* : The normal component of the velocity on the hull surface must be zero. So the boundary conditions for the first and second order approximations can be written from Equation (1) as

$$\begin{aligned}&\varepsilon : \ \nabla \phi_1 \cdot n = -U n_x \\ &\varepsilon^2 : \nabla \phi_2 \cdot n = 0\end{aligned} \tag{3}$$

in which $n = n_x i + n_y j + n_z k$ denotes the unit normal vector on the surface and is positive into the fluid.

(ii) *Free surface condition*: The kinematic and dynamic boundary conditions on the free surface can be written as:

$$\Phi_x \zeta_x + \Phi_y \zeta_y - \Phi_z = 0 \qquad \text{at} \quad z = \zeta \tag{4}$$

$$g\zeta + \frac{1}{2}(\nabla\Phi \cdot \nabla\Phi - U^2) = 0 \qquad \text{at} \quad z = \zeta \tag{5}$$

Combining Equations (4) and (5) we get

$$\nabla\Phi . \nabla\left[\frac{1}{2}(\nabla\Phi \cdot \nabla\Phi)\right] + g\Phi_z = 0 \ \text{ at } \ z = \zeta \tag{6}$$

The free surface boundary condition (6) is nonlinear in nature and should be satisfied on the actual surface, which is unknown and can be linearized as a part of the solution using the perturbation method. Substituting Equation (1) into Equation (6) and expanding the potential ϕ in a Taylor series about the mean free surface z = 0 the following first and second order free surface boundary conditions can be obtained as

$$\begin{aligned}&\varepsilon \ : \ \phi_{1xx} + K_0 \phi_{1z} = 0 \\ &\varepsilon^2 : \ \phi_{2xx} + K_0 \phi_{2z} = R_2\end{aligned} \qquad \text{at} \quad z = 0 \tag{7}$$

where, K_0 (= g/U^2) is the wave number and

$$R_2 = -\frac{1}{U}\frac{\partial}{\partial x}\left(\phi_{1x}^2 + \phi_{1y}^2 + \phi_{1z}^2\right) - \zeta_1 \frac{\partial}{\partial z}\left(\phi_{1xx} + K_0 \phi_{1z}\right)$$

(iii) The wake surface S_w is assumed to have zero thickness. The pressure jump across S_w is zero, while a jump in the potential is allowed.

$$(\Delta p)_{\text{on } S_w} = p^+ - p^- = 0$$

$$\Delta\left(\frac{\partial\phi}{\partial n}\right)_{\text{on } S_W} = \left(\frac{\partial\phi}{\partial n}\right)^{+} - \left(\frac{\partial\phi}{\partial n}\right)^{-} = 0$$

(iv) For the steady lifting problem, the potential jump across the wake surface is the same as the circulation around the body and is constant in the stream wise direction.

$$(\Delta\phi)_{\text{on } S_W} = \phi^{+} - \phi^{-} = \Gamma$$

A Kutta condition is required at the trailing edge to uniquely specify the circulation. In its most general form, it states that the flow velocity at the trailing edge remains bounded.

$$\left|\nabla\phi\right|_{TE} < \infty$$

(v) Radiation condition: It is necessary to impose a condition to ensure that the free surface waves vanish upstream of the disturbance.

3. Wave profile and resistance

The first and second order contribution to the linearized equation of wave profile can be obtained by Taylor's series expansion from Equations (1) and (5) as

$$\zeta_1 = -\frac{U}{g}\phi_{1x} \tag{8}$$

$$\zeta_2 = -\frac{U}{g}\phi_{2x} - \frac{U}{g}\zeta_1\phi_{1xz} - \frac{1}{2g}\left(\phi_{1x}^2 + \phi_{1y}^2 + \phi_{1z}^2\right) \tag{9}$$

Now the linearized equation of the second order wave profile can be written as

$$\zeta = \zeta_1 + \zeta_2 \tag{10}$$

After calculating the fluid velocity $\nabla\Phi$ on the control points on the hull surface the pressure co-efficient can be evaluated as:

$$C_p = 1 - \left(\frac{\nabla\Phi}{U}\right)^2$$

$\nabla\Phi = U + \nabla\phi_1$ for first order approximation

$\nabla\Phi = U + \nabla\phi_1 + \nabla\phi_2$ for second order approximation

Now including the waterline integral the wave- making coefficient can be obtained as

$$C_w = -\frac{\sum_{i=1}^{N_H} C_p n_x \Delta S}{\sum_{i=1}^{N_H} \Delta S} - \frac{\rho g \oint_{WL} \zeta^2 n_x dl}{\rho S U^2} \tag{11}$$

where, ΔS denotes the area of a panel on the hull surface.

4. Result and discussion

The numerical algorithms outlined in the preceding section have been applied to various configurations of pentamaran in order to analyze the hydrodynamic characteristics at various speeds. The selected multihull ships are composed of Wigley hull and the equation of this mathematical hull surface is as follows:

$$y = \frac{1}{2}B\left(1-\frac{4x^2}{L^2}\right)\left(1-\frac{z^2}{T^2}\right)$$

where , L, B and T are the vessel length, width and draft respectively. A pentamaran is composed of the central main hull being two times longer than the outriggers. They all have the dimensions of L/B = 10 and B/T = 1.6. The stagger L_x is defined as the longitudinal distance between midsection of the central main hull and the outriggers, positive for outriggers towards the stern. The spacing L_y is defined as the lateral distance between the center plane of the main hull and that of the outriggers. These two parameters are symmetrically varied which is defined in Table 1.

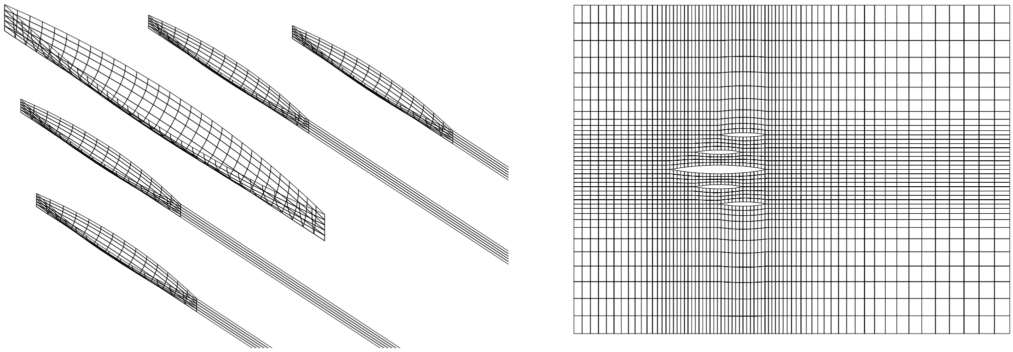
Table 1. Various configurations of Wigley pentamaran

	Case 1		Case 2		Case 3		Case 4		Case 5	
	L_X	L_Y	L_X	L_Y	L_X	L_Y	L_X	L_Y	L_X	L_Y
Main hull	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Outrigger 1	-0.25	-0.2	-0.125	-0.2	-0.25	-0.2	0.0	-0.2	0.0	-0.2
Outrigger 2	-0.25	0.2	-0.125	0.2	-0.25	0.2	0.0	0.2	0.0	0.2
Outrigger 3	0.25	-0.4	0.125	-0.4	0.0	-0.4	0.125	-0.4	0.0	-0.4
Outrigger 4	0.25	0.4	0.125	0.4	0.0	0.4	0.125	0.4	0.0	0.4

The paneled region of the free surface is extended about 1.5 of the ship length upstream, 3.0 of the ship length downstream from the body and 1.8 of the ship length sideward of the ship. Due to the symmetry, only a half of the pentamaran and free surface is taken into account in the computation scheme. The effect of the remaining half of the computational domain is incorporated while calculating the influence coefficient by the reflection of the discretized elements. The pentamaran hull is discretized by 450 (30×5+4×15×5) panels and the free surface by 1050 (70×15) panels. The wake surface behind the hulls is discretized by 5×5 panels. The panel arrangement for the pentamaran model is given in Fig. 1. To better resolve complex flow features around the multihull the grid is stretched in all directions with a concentration of grid points near the hull and in the stern region where the flow changes rapidly.

Dawson [11] points out that a two-point finite difference operator used in the free surface condition results in waves that are strongly damped downstream from a disturbance and three-point operator is better than two-point operator. In the present numerical scheme two-point upstream finite difference operator is used in a portion of the free surface having concentrated grid and three-point is used in the rest part of the free surface in order to satisfy the radiation condition.

The computed first and second order wave-making resistance coefficients C_w of pentamaran hull of different configurations are compared in Figs. 2 to 5 respectively. At low speeds the waves made by the ship very small and the resistance is almost frictional. With further increase in speed, the value of C_w begins increase more and rapidly up to a certain Froude number. In case of pentamaran (case 1 & 2 configurations) the humps and hollows of C_w curves in Figs. 2 & 3 appears to be the same locations but some discrepancies are found for the pentamaran of case 5 configurations. From these it can also be observed that the wave resistance coefficients of pentamaran are very sensitive to the longitudinal position of the outriggers.



(a) Body and wake surface

(b) Free surface

Fig 1. Panel arrangement of the pentamaran model (Case 4)

Figures 6 and 7 show the first and second order wave profile on the main hull of the pentamaran of various configurations at different Froude numbers.

The wave pattern generated by a pentamaran that is moving at a constant speed in deep water is depicted in Fig. 8. Generally, there is a symmetrical set of diverging waves that moves obliquely out from the vessel's sailing line and a set of transverse waves that propagate along the sailing line. The transverse and diverging waves meet along the cusp locus lines that form an angle of $19^0 28'$ with the sailing line. The largest wave height is found where the transverse and diverging waves meet. If the speed of the vessel is increased, this wave crest pattern retains the same geometric form, but expands in size as the individual wave lengths increase.

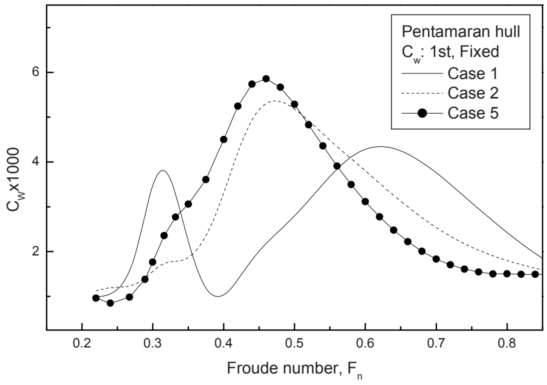


Fig. 2. Stagger effect on the first order wave making resistance on the pentamaran hull

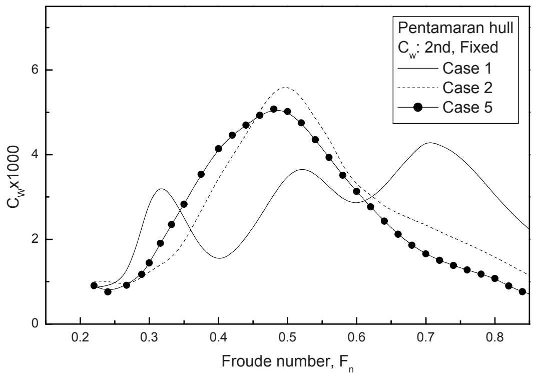


Fig. 3. Stagger effect on the second order wave making resistance on the pentamaran hull

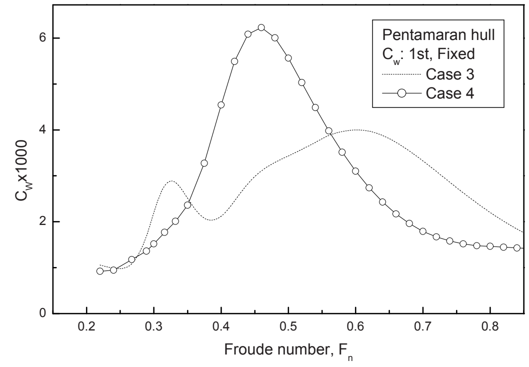


Fig. 4. Stagger effect on the first order wave making resistance on the pentamaran hull

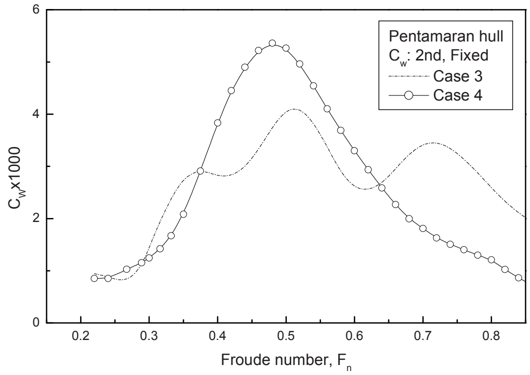
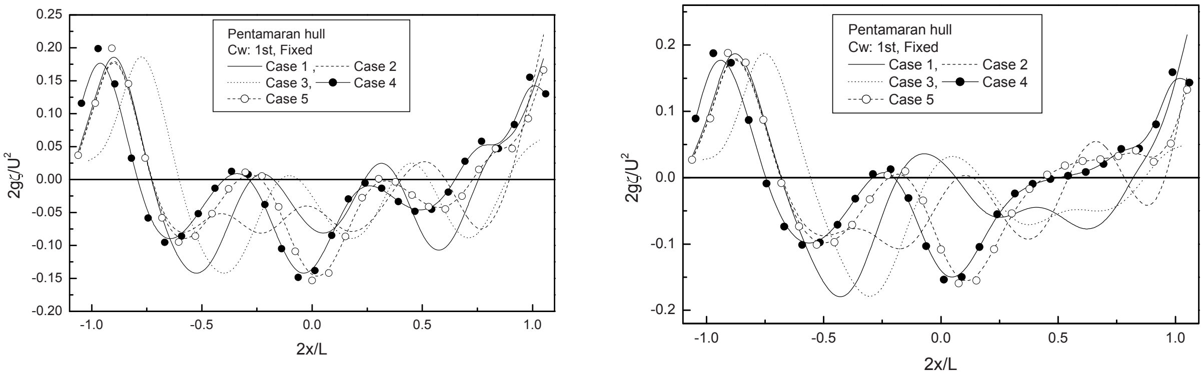
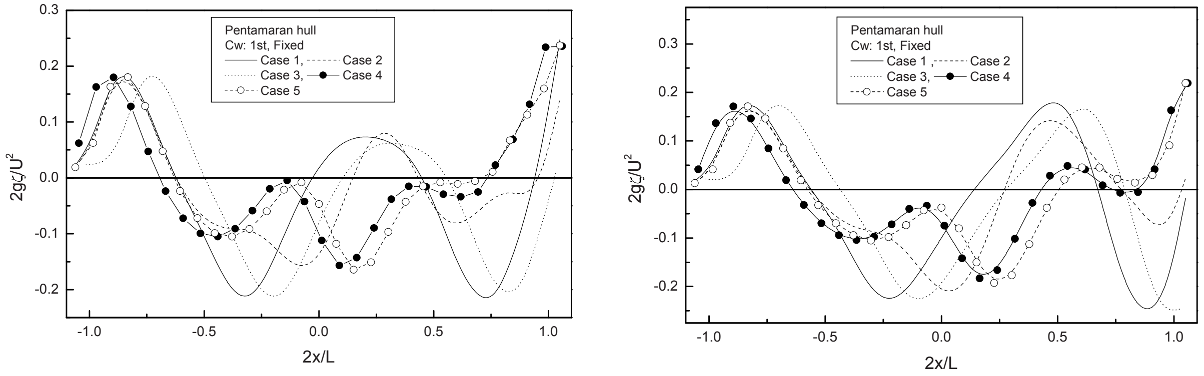


Fig. 5. Stagger effect on the second order wave making resistance on the pentamaran hull



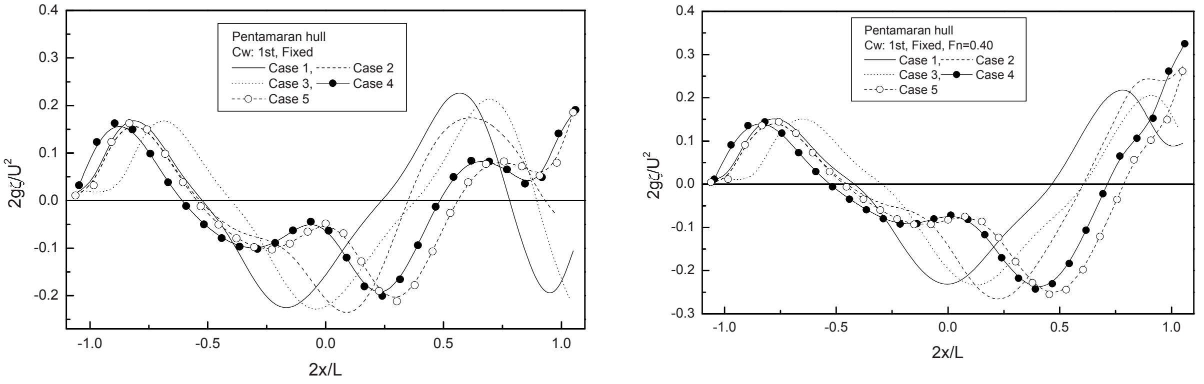
(a) Wave Elevation at $F_n = 0.240$

(b) Wave Elevation at Fn = 0.267



(c) Wave Elevation at $F_n = 0.300$

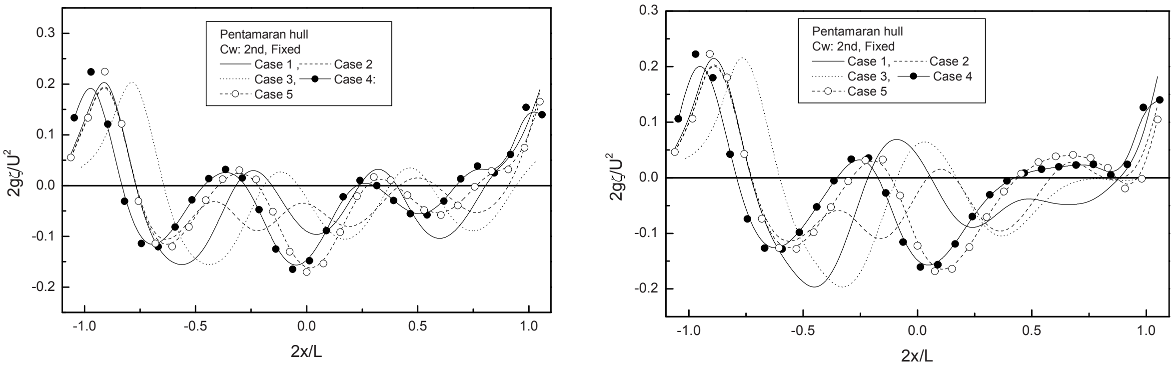
(d) Wave Elevation at $F_n = 0.332$



(e) Wave Elevation at $F_n = 0.350$

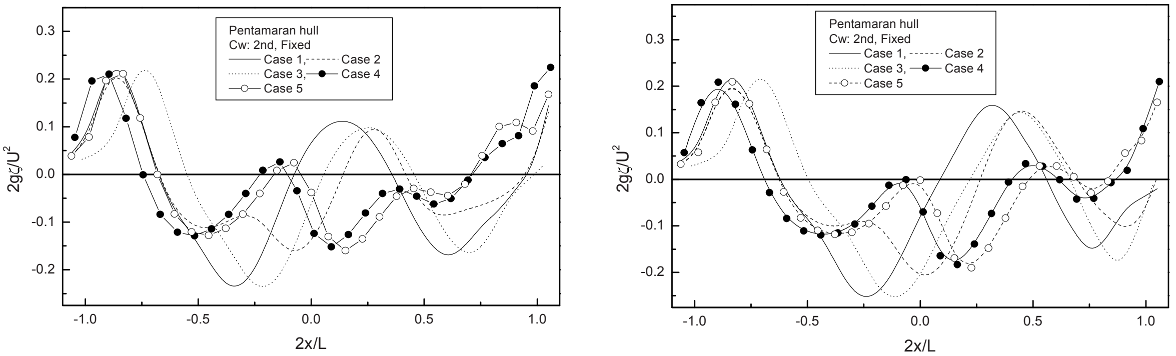
(f) Wave Elevation at $F_n = 0.400$

Fig. 6. Wave pattern (1st order, Fixed) at various Froude numbers for the pentamaran of different configurations



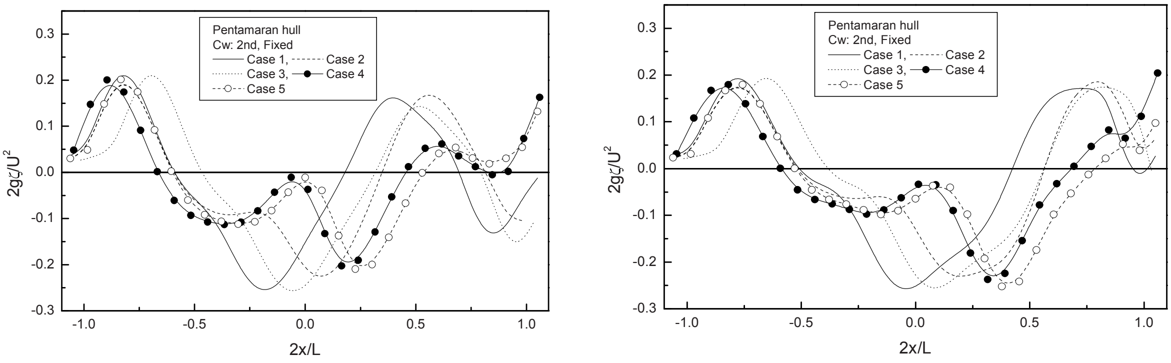
a) Wave Elevation at $F_n = 0.240$

(b) Wave Elevation at $F_n = 0.267$



(c) Wave Elevation at $F_n = 0.300$

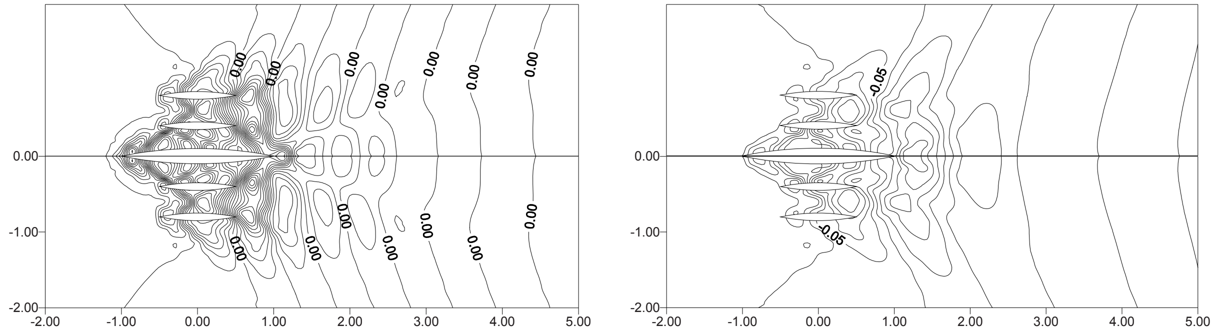
(d) Wave Elevation at $F_n = 0.332$



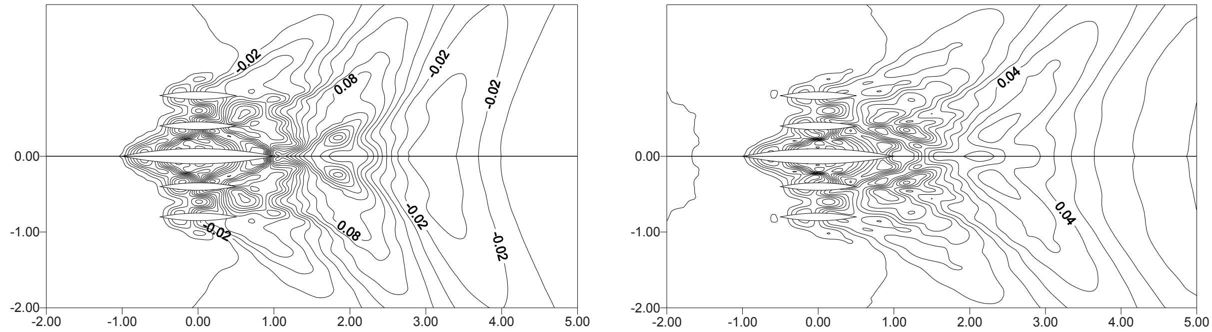
(e) Wave Elevation at $F_n = 0.350$

(f) Wave Elevation at $F_n = 0.400$

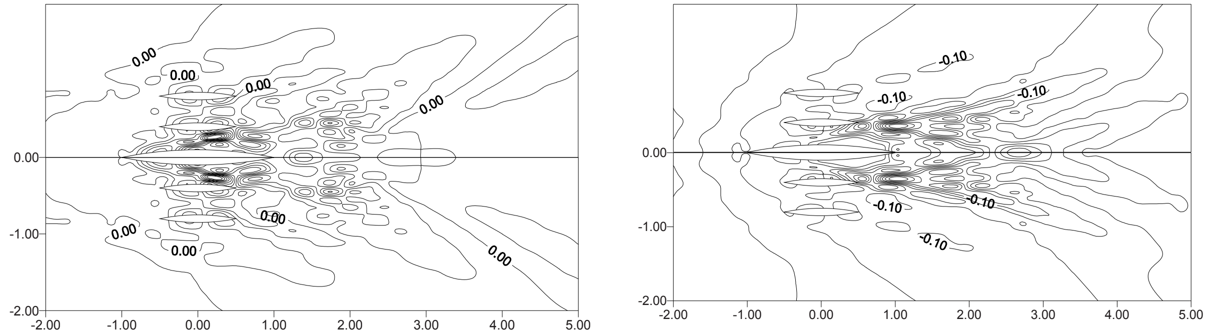
Fig. 7. Wave pattern (2^{nd} order, Fixed) at various Froude numbers for the pentamaran of different configurations



(a) Wave pattern $F_n = 0.3$ (b) Wave pattern $F_n = 0.4$



(c) Wave pattern $F_n = 0.5$ (d) Wave pattern $F_n = 0.6$



(e) Wave pattern $F_n = 0.8$ (f) Wave pattern $F_n = 1.0$

Fig. 8. Wave pattern (2nd, Fixed) of the pentamaran hull at various Froude number for case 5

5. Conclusions

This paper successfully presents a potential based boundary element method in connection with Kelvin classical linearized free surface condition for predicting the hydrodynamic characteristics of pentamaran hull moving in unbounded water with a constant speed. The following conclusions can be drawn from the present numerical analysis:

(i) The present method could be a useful tool for the hydrodynamic analysis of the potential flow around the multihull at an infinite depth of water and may also achieve a high precision by taking smaller panels on some regions of the free surface where the flow changes rapidly.

(ii) At higher speeds having Froude number greater than 0.8 the interference among the monohulls of the pentamaran are very small and the wave-making resistance can be reduced significantly.

(iii) Each computational method is dependent on the grid and on various other parameters. So, the present technique must be validated before it may be applied to the complicated flow around such hull shapes.

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